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Vibration, Deflections

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1 Introduction

The fact, that footbridges – independent of material – are likely to vibrate, is still actual. But there is a practical method to handle it. This method is presented here and is part of the draft of Eurocode 5-2 [1].

2 Designing the footbridge in ultimate limit state and deflections

2.1 Designing the footbridge in the ultimate limit state

Designing the footbridge in the ultimate limit state only leads to low frequencies. This is shown in [2] for steel bridges, but transferable to all materials.

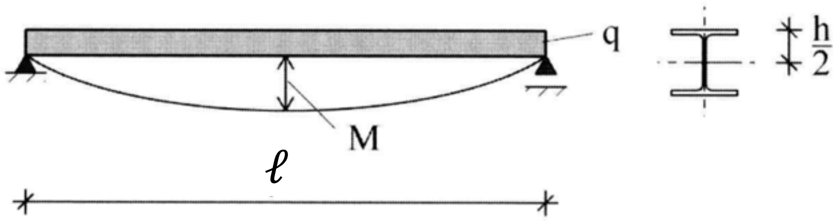

Figure 1: Bending moment of beam with constant load q, taken from [2]

The maximum bending moment of the girder is:

$$M_d = \gamma_{G/Q} \cdot \frac{q \cdot \ell^2}{8}$$

The maximum bending stress is:

$$\sigma_{m,d} = \frac{M_d}{W} = \frac{M_d \cdot h}{2 \cdot I}$$

In case of 100 % exploitation of the material, the maximum bending stress is equal to the strength.

$$\sigma_{m,d} = f_{m,d} = \frac{f_{m,k} \cdot k_{mod}}{\gamma_{m,timber}} or \frac{f_{y,k}}{\gamma_{m,steel}}$$

This leads to:

$$\frac{M_d \cdot h}{2 \cdot I} = \gamma_{G/Q} \cdot \frac{q \cdot \ell^2}{8} \frac{h}{2 \cdot I} = \frac{\gamma_{G/Q} \cdot q \cdot \ell^2 \cdot h}{16 \cdot I} = \frac{f_{m,k} \cdot k_{mod}}{\gamma_{m,timber}}$$

From this, the required area moment of intertia $I_{req,ULS}$ can be calculated, based on the ultimate limit state only. It is assumed that $\gamma_{G/Q} \approx 1{,}45;\ k_{mod} = 0{,}9;\ f_{m,k} = 24 \cdot 10^6 \frac{N}{m^2}$; see GL24h.

$$I_{req,ULS} = \frac{\gamma_{G/Q} \cdot \gamma_{m,timber} \cdot q \cdot \ell^2 \cdot h}{16 \cdot f_{m,k} \cdot k_{mod}} = \frac{1{,}45 \cdot 1{,}3 \cdot q \cdot \ell^2 \cdot h}{16 \cdot 24 \cdot 10^6 \cdot 0{,}9} = 5{,}45 \cdot 10^{-9} \cdot q \cdot \ell^2 \cdot h$$

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2.2 Designing the footbridge to deflection criteria

Designing the footbridge to deflection criteria leads to the following:

The maximum deflection of a simple supported beam with constant load q is:

$$w = \frac{5}{384} \cdot \frac{q \cdot \ell^4}{EI}$$

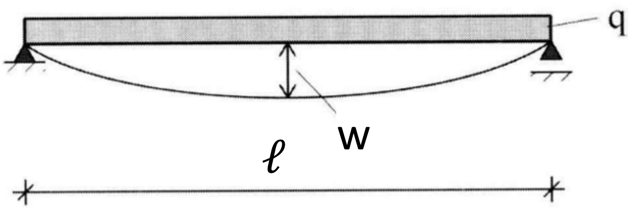

Figure 2: Deflection of beam with constant load q, taken from [2]

The limit of deflection is taken as $\frac{\ell}{x}$ and later as $\frac{\ell}{500}$.

This leads to the required area moment of inertia regarding the deflection criteria, with $E = 11500 \cdot 10^6 \frac{N}{m^2}$; see GL24h.

$$I_{req,Defl} = \frac{5}{384} \cdot \frac{q \cdot \ell^4 \cdot x}{E \cdot \ell} = \frac{5}{384} \cdot \frac{q \cdot \ell^3 \cdot 500}{11500 \cdot 10^6} = 5{,}66 \cdot 10^{-10} \cdot q \cdot \ell^3$$

2.3 Calculating the frequency

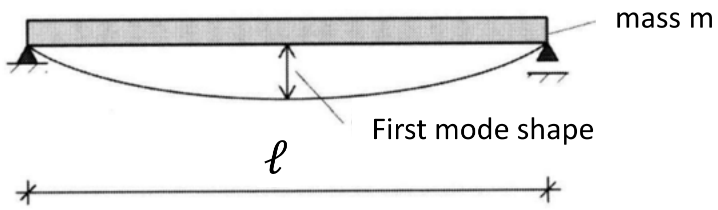

Figure 3: First mode shape of beam with constant mass m, taken from [2]

The frequency corresponding to the first mode shape of a simply supported beam can be calculated as:

$$f = \frac{\pi}{2 \cdot \ell^2} \cdot \sqrt{\frac{EI}{m}}$$

As the mass is taken to $m = \frac{q}{g}$, f is the frequency of the fully loaded bridge. $g = 9{,}81 \frac{m}{s^2}$ is the gravitational acceleration.

$$f^2 = \frac{\pi^2}{4 \cdot \ell^4} \cdot \frac{EI \cdot g}{q} = \frac{\pi^2}{4 \cdot \ell^4} \cdot \frac{11500 \cdot 10^6 \cdot I \cdot 9{,}81}{q} = \frac{278 \cdot 10^9 \cdot I}{\ell^4 \cdot q}$$

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2.4 Combining the frequency and the required area moment of inertia

Replacing $I_{req,ULS}$ in this formula leads to the following frequencies:

$$f_{req,ULS} = \sqrt{\frac{278 \cdot 10^9 \cdot I}{\ell^4 \cdot q}} = \sqrt{\frac{278 \cdot 10^9 \cdot 5{,}45 \cdot 10^{-9} \cdot q \cdot \ell^2 \cdot h}{\ell^4 \cdot q}} = 38{,}9 \cdot \frac{\sqrt{h}}{\ell}$$

Assuming a beam with 1,5m height: $f_{ULS\_1,5m} = 38{,}9 \cdot \frac{\sqrt{1{,}5}}{\ell} = 47{,}6 \cdot \frac{1}{\ell}$

Assuming a beam with 1,0m height: $f_{ULS\_1,0m} = 38{,}9 \cdot \frac{\sqrt{1{,}0}}{\ell} = 38{,}9 \cdot \frac{1}{\ell}$

Assuming a beam with 0,5m height: $f_{ULS\_0,5m} = 38{,}9 \cdot \frac{\sqrt{0{,}5}}{\ell} = 27{,}5 \cdot \frac{1}{\ell}$

Replacing $I_{req,Defl}$ in this formula leads to the following frequencies:

$$f_{Defl} = \sqrt{\frac{278 \cdot 10^9 \cdot I}{\ell^4 \cdot q}} = \sqrt{\frac{278 \cdot 10^9 \cdot 5{,}66 \cdot 10^{-10} \cdot q \cdot \ell^3}{\ell^4 \cdot q}} = 12{,}5 \cdot \frac{1}{\sqrt{\ell}}$$

The graph in figure 4 shows the interaction between designing for ultimate limit state or deflection and the resulting natural frequency for the fully loaded bridge.

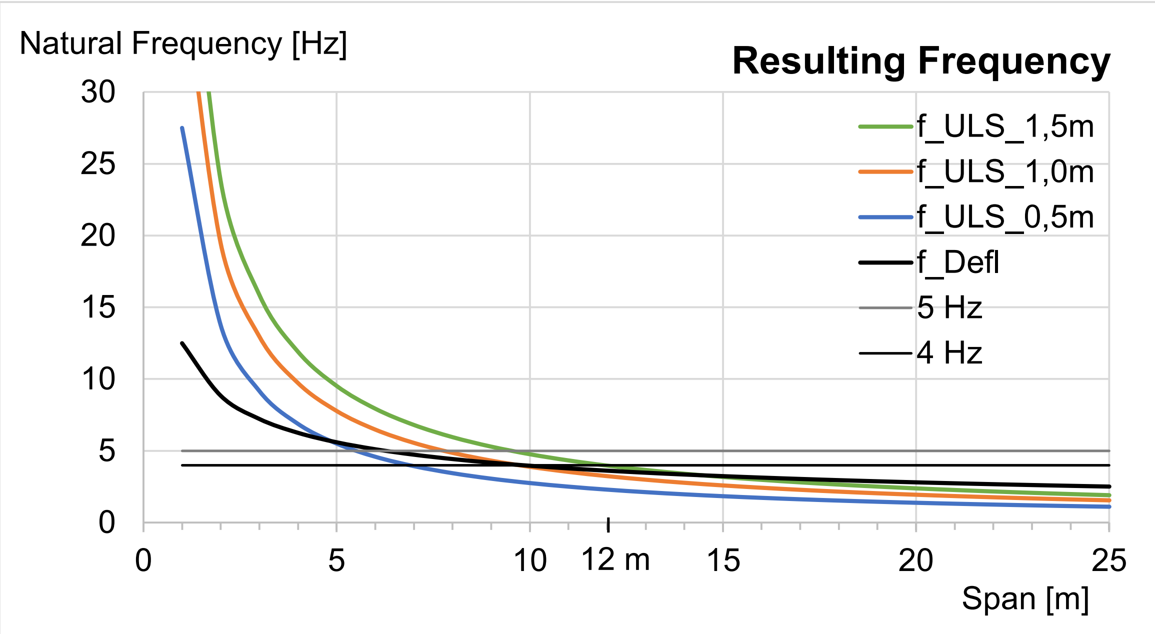

Figure 4: Resulting Frequency when designing the footbridge to ultimate limit state or deflection criteria

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3 Method suggested in prEN 1995-2:202x (E). Eurocode 5 — Design of Timber Structures — Part 2: Bridges

3.1 Most of the footbridges are in resonance

Figure 4 shows, that most of the bridges with spans greater than (7m –) 12 m have natural frequencies lower than 5-Hz, or lower than 4 Hz, as the fully loaded bridge was based on the calculations above.

What does this mean? This means, that most of the bridges can be excited in resonance by the first or second harmonic part of the dynamic load to the underground while walking. Figure 5 shows, how to handle the low frequency footbridges in general: If the proof of vibration is not fulfilled, the installation of a tuned mass damper should be planned and regarded in the design of the bridge and in the costs. After the construction of the bridge, a vibration measurement can help to decide, whether the installation of the tuned mass damper is necessary or not.

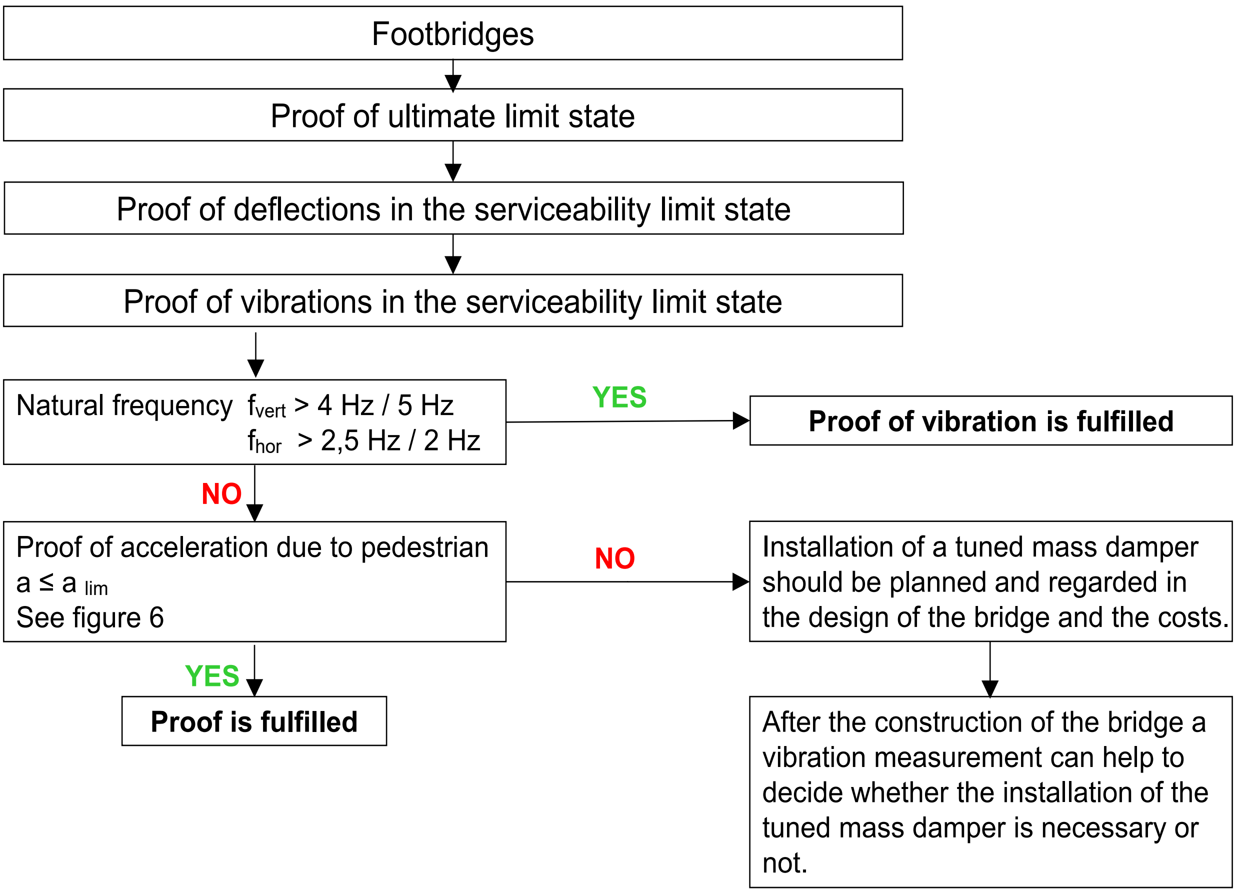

Figure 5: Flowchart how to handle low frequency footbridges in general, see [1], [3] and [4]

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3.2 Calculation of the acceleration of the bridge in resonance

Eurocode 5-2 [1] gives formulas, how to calculate the resulting acceleration of the footbridge, when the bridges is excited in resonance. Table H.1 in Eurocode 0 [5] gives different limit values, which can be chosen for the limit acceleration depending on different comfort levels.

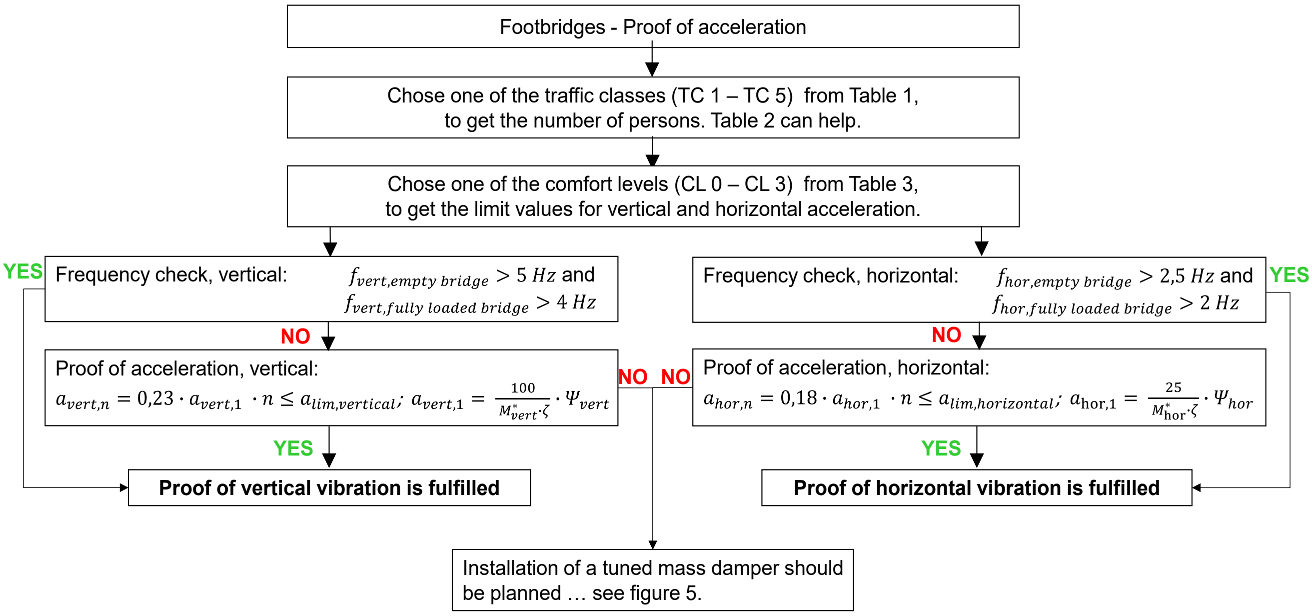

Figure 6: Flowchart of the simplified model to proof the acceleration of footbridges, see [1] and [6]

Table 1: Traffic Classes (move out from Table G.1, Eurocode 1 [6])

Traffic Class	Description	Explanation	Pedestrian Stream, given by density d [P/m²]
TC 1	Very weak traffic	Seldom used footbridge; bridge built to link sparsely populated areas	0.1
TC 2	Weak traffic	Footbridge for standard use; bridge that may occasionally be crossed by large groups of people but that will never be loaded throughout its bearing area	0.2
TC 3	Dense traffic	Urban footbridge linking up populated areas; bridge subjected to road traffic and that may occasionally be loaded throughout its bearing area	0.5
TC 4	Very dense traffic	Urban footbridge linking up high pedestrian density areas; bridge located for instance, nearby a rail or underground station	1.0
TC 5	Exceptionally dense traffic	Urban footbridge linking up exceptionally high pedestrian density areas; bridge located for instance, nearby an arena that may accommodate a large number of spectators	1.5

Normally no dynamic analyses are required for footbridges in traffic class TC1. For very light footbridges, e.g. girder-type bridges with pavement consisting of light material, it is advised to select at least TC2 to ensure a minimum amount of comfort. Very light footbridges may show high accelerations without any resonance.

The weight of a single person is 0.8 kN according to Annex A, Eurocode 0 [5].

The owner of the bridge should specify the appropriate traffic class, based on the level of pedestrian traffic that the bridge is expected to be subjected to. Table 2 can help.

Table 2: Recommended traffic classes for bridges according to their location

Location of the bridge	**Frequency of use**	**Recommended traffic class**
Outside of towns	sometimes	TC 1
	often	TC 2
Inside of towns	often	TC 2
In areas with huge events	often	TC 3
Close to stations	sometimes	TC 3
	often	TC 4
In areas with stadiums or parks	often	TC 4
On routes of fun runs	often	TC 5

Table 3: Comfort levels and corresponding allowed vertical and horizontal accelerations (move out from Table H.1, Eurocode 0 [5] and [7])

Comfort level	**Degree of comfort**	**Explanation**	**Vertical acceleration** $a_{lim,vertical}$ **[m/s²]**	**Horizontal acceleration** $a_{lim,horizontal}$ **[m/s²]**
CL 3	Maximum	Accelerations practically imperceptible to the users	≤ 0.5	≤ 0.1 ≈ 0.15
CL 2	Medium	Accelerations merely perceptible to the users	≤ 1.0	≤ 0.3
CL 1	Minimum	Accelerations perceived by the users, but not intolerable	≤ 2,5	≤ 0.8 ≈ 0.75
CL 0	No limit set	Accelerations strongly perceived by the users, intolerable by most	> 2,5	> 0.8 ≈ 0.75

M^*_{vert} and M^*_{hor} are the generalized masses of the bridge in kg for vertical or horizontal vibrations as a function of the mass per unit length of the bridge and the distance between the supports. The generalized mass M* may be different for vertical or horizontal accelerations.
The generalized mass of a single span girder or truss system with span ℓ is $M^* = \frac{1}{2} \cdot m \cdot \ell$

m is the mass per unit length of the bridge in kg/m including the permanent load plus the pedestrian load throughout the bridge bearing area, with a number of pedestrians per square meter – depending on the traffic class, see table 1.

ζ is the damping ratio to the relevant natural frequency, see table 4.

ψ_{vert} and ψ_{hor} are the reduction coefficients taking into account the probability that the footfall frequency (both jogging and walking) approaches the critical range of natural frequencies of the bridge, see Figures 7 and 8.

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Table 4: Damping ratio according to Lehr

Static system of the bridge	Damping ratio ζ
cable-stayed bridges with very long cables vibrating themselves	0,5 %
cable-stayed bridges superstructures without mechanical joints timber-concrete-composite superstructures bridges with light-weight timber deck (e.g. roofed bridge with planked timber deck) with main girders with mechanical joints	1,0 %
superstructures with mechanical joints timber bridges with a heavy bridge deck (e.g stress laminated decks or block-glued decks) with a floating layer; e.g. a sealant system	1,5 %
bridge system consisting of a heavy deck supported by arches or trusses superstructures with a floating layer (e.g. a rear-ventilated road-surface)	2,0 %

In case of horizontal vibration of trough bridges (without torsion connection) with a lateral wind bracing (built with fasteners) the damping ratio values given above may be doubled.

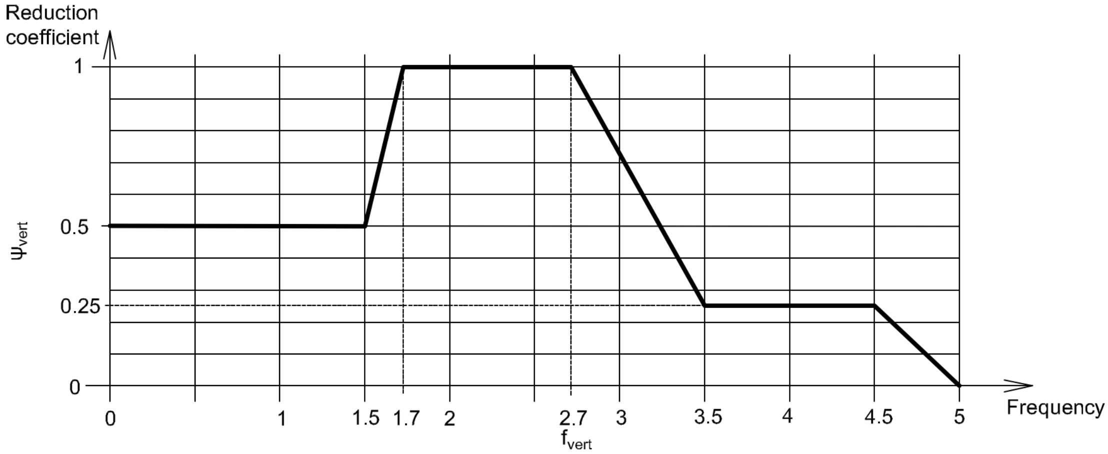

Figure 7: Relationship between the vertical natural frequency f_{vert} *and the reduction coefficient* ψ_{vert}*, taken from [1]*

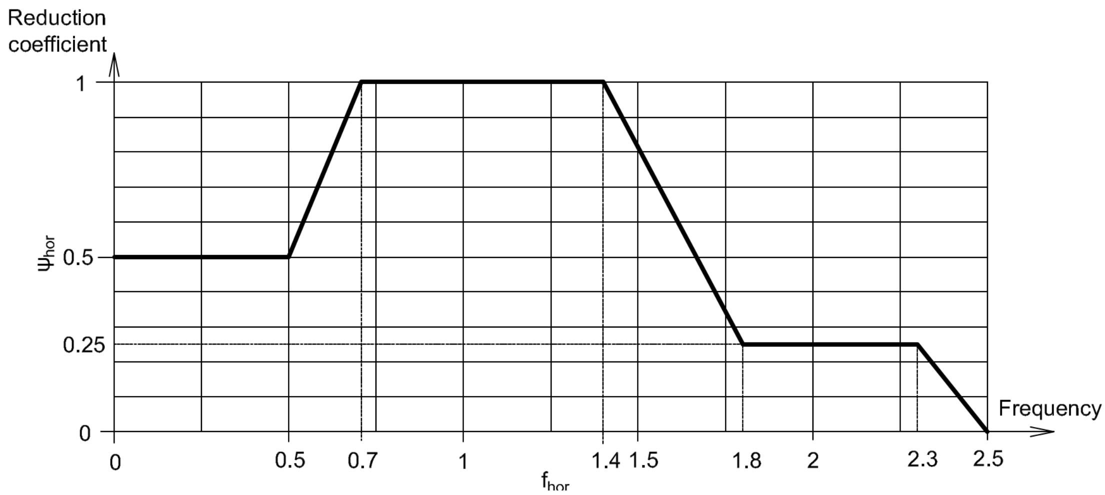

Figure 8: Relationship between the horizontal natural frequency f_{hor} *and the reduction coefficient* ψ_{hor} *taken from [1]*

4 Background

4.1 How are the higher harmonics considered?

The higher harmonics, namely the second harmonic are considered by the reduction coefficient. As the Fourier coefficient of the second harmonic (α_2 =0,1 – 0,2) is about one-half to one fourth of the first harmonic (α_1 =0,4), the reduction coefficient is $\Psi = 0{,}25$ in the range of two times step frequency ($2 \cdot f_S$ =2,5 – 5,0 Hz) – compared to $\Psi = 1$ in the range of step frequency (f_S =1,25 – 2,5 Hz), see Eurocode 1 [6], table G.2. Synchronisation with the second harmonic is rather unlikely, therefore $\Psi = 0{,}25$ is on the save side.

4.2 How are joggers considered?

The load function of runners (= joggers) has different Fourier coefficients compared to walkers. The main difference lies in the first harmonic, as the first Fourier coefficient for jogging ($\alpha_{1,jogging}$ =1,5) is about 4 times the Fourier coefficient of walking ($\alpha_{1,walking}$ =0,4). This leads to the following acceleration-relation due to 1 person: $a_{vert,1,jogging} = 4 \cdot a_{vert,1,walking}$

Eurocode 1 [6], table G.1 gives the number of persons, which should be regarded, see table 5:

Table 5: Number of persons in a pedestrian group (walking) and in a jogging group, taken from Eurocode 1 [6]

Pedestrian group n_w	Jogging group n_j	Relation n_w/n_j
1	0	-
2	0	-
4	1	4
8	2	4
16	4	4

One can see that the numbers of persons in a walking group is at least four times the number of persons in a jogging group. Therefore, the factor 4 for higher load while jogging cancels out the factor 4 for the bigger group while walking.

Figures 9 and 10 show the correlation. One can see that the black curves in figures 7 and 8 are the envelope curves of the mentioned cases “higher harmonics” and “joggers”.

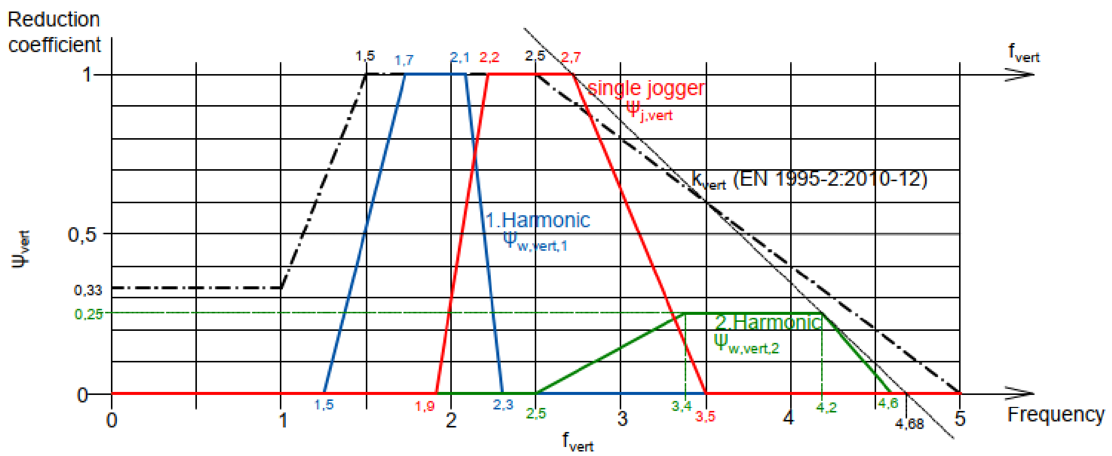

Figure 9: Relationship between the vertical natural frequency f_{vert} and the reduction coefficient $\psi_{w,vert}$ (walking) and $\psi_{j,vert}$ (jogging), taken from [11]

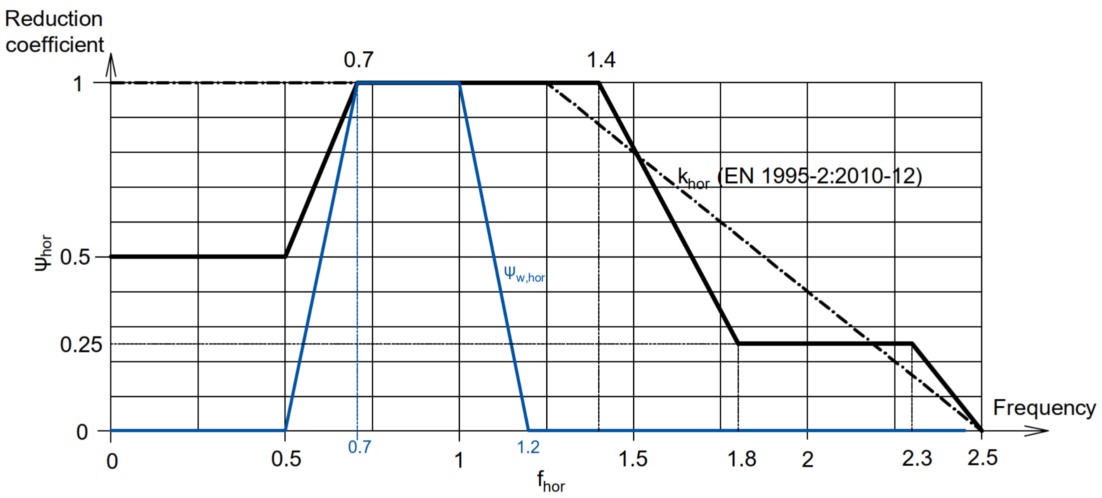

Figure 10: Relationship between the horizontal natural frequency f_{hor} and the reduction coefficient ψ_{hor} (both walking and jogging), taken from [11]

4.3 Why are pedestrian groups not regarded in the simplified model?

The simplified model builds on the load model pedestrian stream, given by a density d [P/m²], see table 1. The maximum number of persons in a pedestrian group is 16. This maximum number is rather equal to the minimum number for the pedestrian stream given in footnote 3, Annex G, Eurocode 1 [6]: “$n \geq 15$ shall be assumed”.

4.4 Why could there be a significant discrepancy between cold and warm weather?

When asphalt is used as paving, natural frequencies can vary depending on temperature. In cold weather, the asphalt becomes stiffer; this can lead to a higher frequency in cold weather. This can appear in both directions, for vertical and horizontal frequencies, see [10].

4.5 Is the simplified model applicable for all bridge types?

The simplified model may be adopted for any bridge type, provided that adequate generalized mass is adopted in the calculations.

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4.6 How to do the advanced proof?

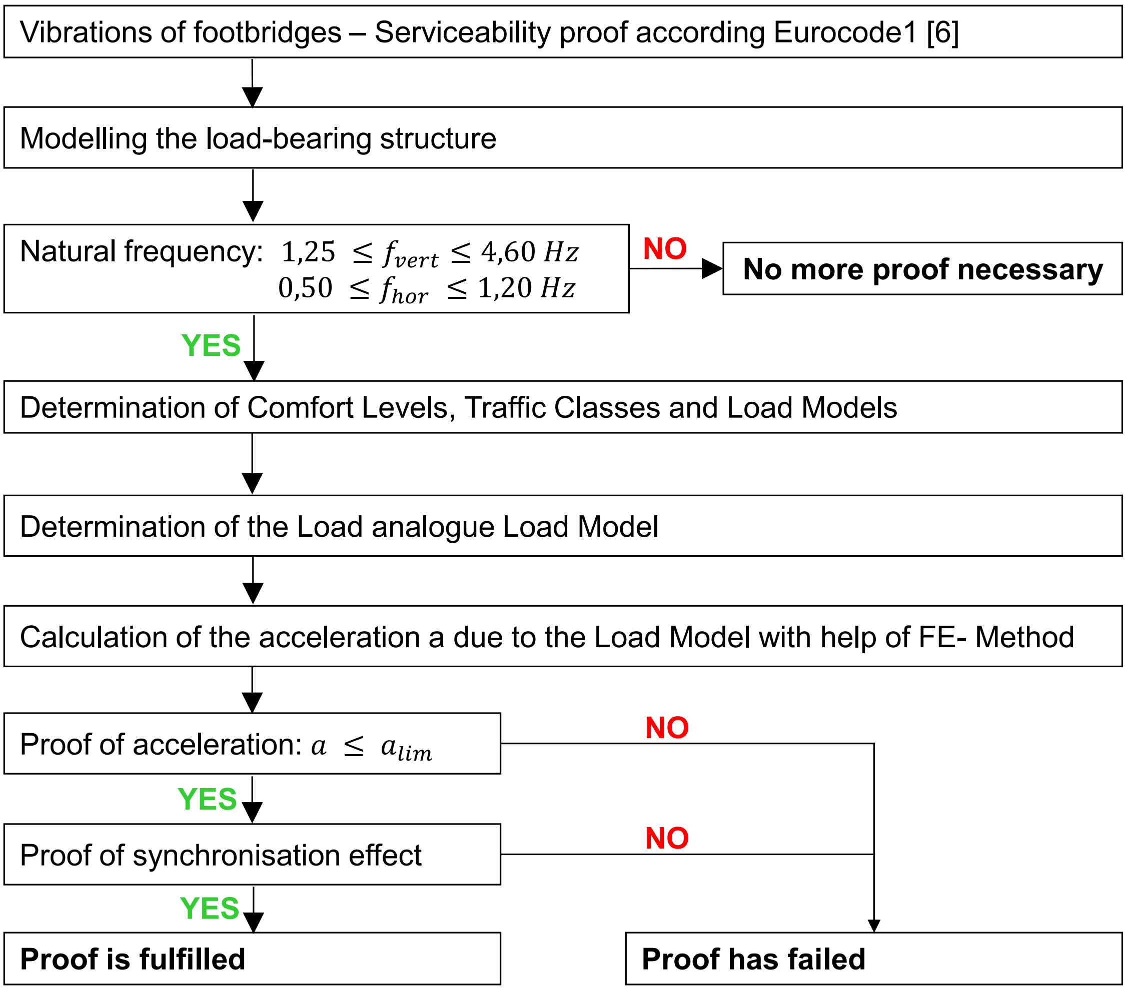

Figure 11: Flowchart for the advanced proof of vibration of footbridges according to Eurocode 1 [6], taken from [4]

4.7 Why are vibration analyses normally not required for foot and bicycle bridges with less than 12 m in span?

Figure 4 shows the correlation between the span of the bridge and the resulting natural frequency, depending on the design of the bridge. It shows that bridges with spans more than 12 m have quite likely frequencies less than 4 Hz (–5-Hz). Bridges with spans less than 12m have natural frequencies greater than 4 Hz (– 5 Hz). Therefore, there is no risk of resonance between step frequency (or twice step frequency) and natural frequency. Nevertheless, it should be kept in mind that very light-weight structures can vibrate in a manner that it significantly reduces the comfort of the pedestrians using the bridge- even without resonance.

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