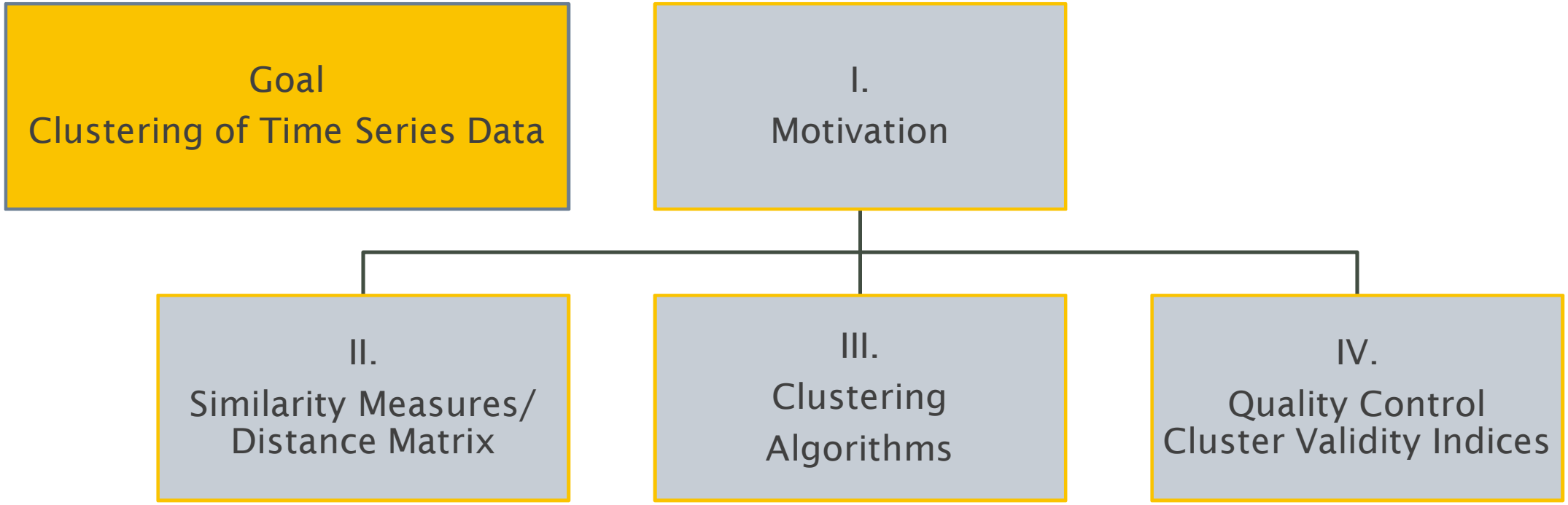
Berner Fachhochschule
Haute école spécialisée bernoise
Bern University of Applied Sciences

A Project with the Federal Office for the Environment FOEN

Clustering of Time Series Data –
Clustering Temperature Measurements of Swiss Rivers

Dr. Vidushi Bigler-Maillart

Structure


Goal
Clustering of Time Series Data
I.
Motivation
II.
Similarity Measures/
Distance Matrix
III.
Clustering
Algorithms
IV.
Quality Control
Cluster Validity Indices

Structure

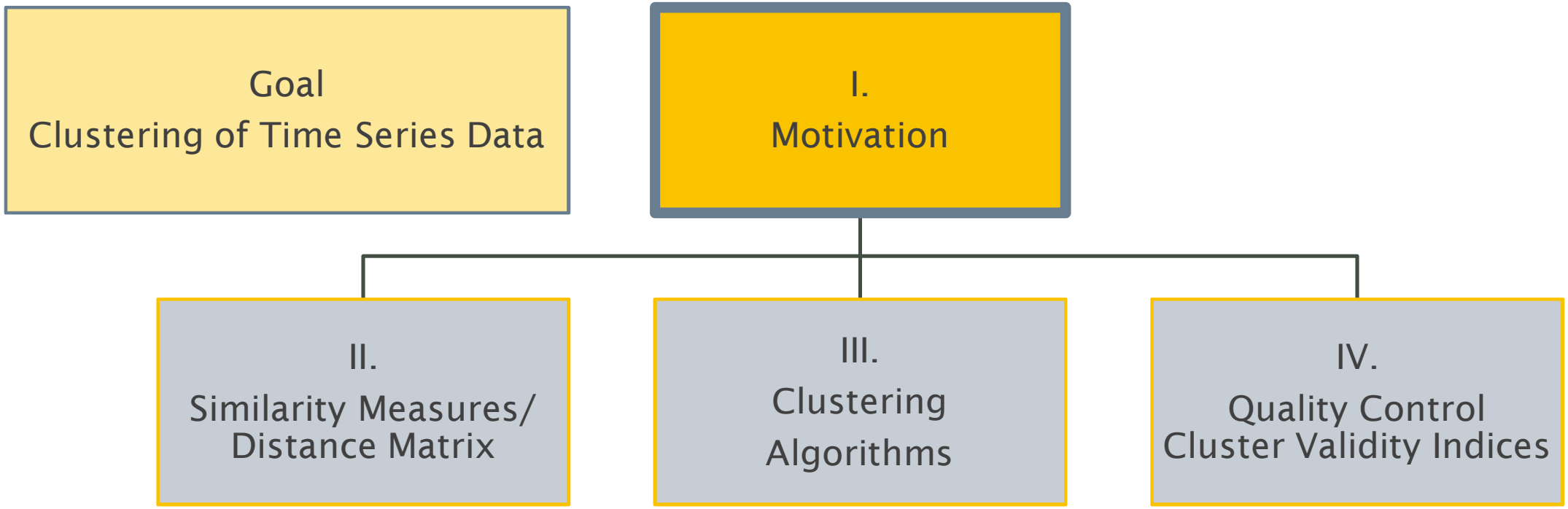
Goal
Clustering of Time Series Data

I.
Motivation

II.
Similarity Measures/
Distance Matrix

It's better to uncover a little than to cover a lot.

Structure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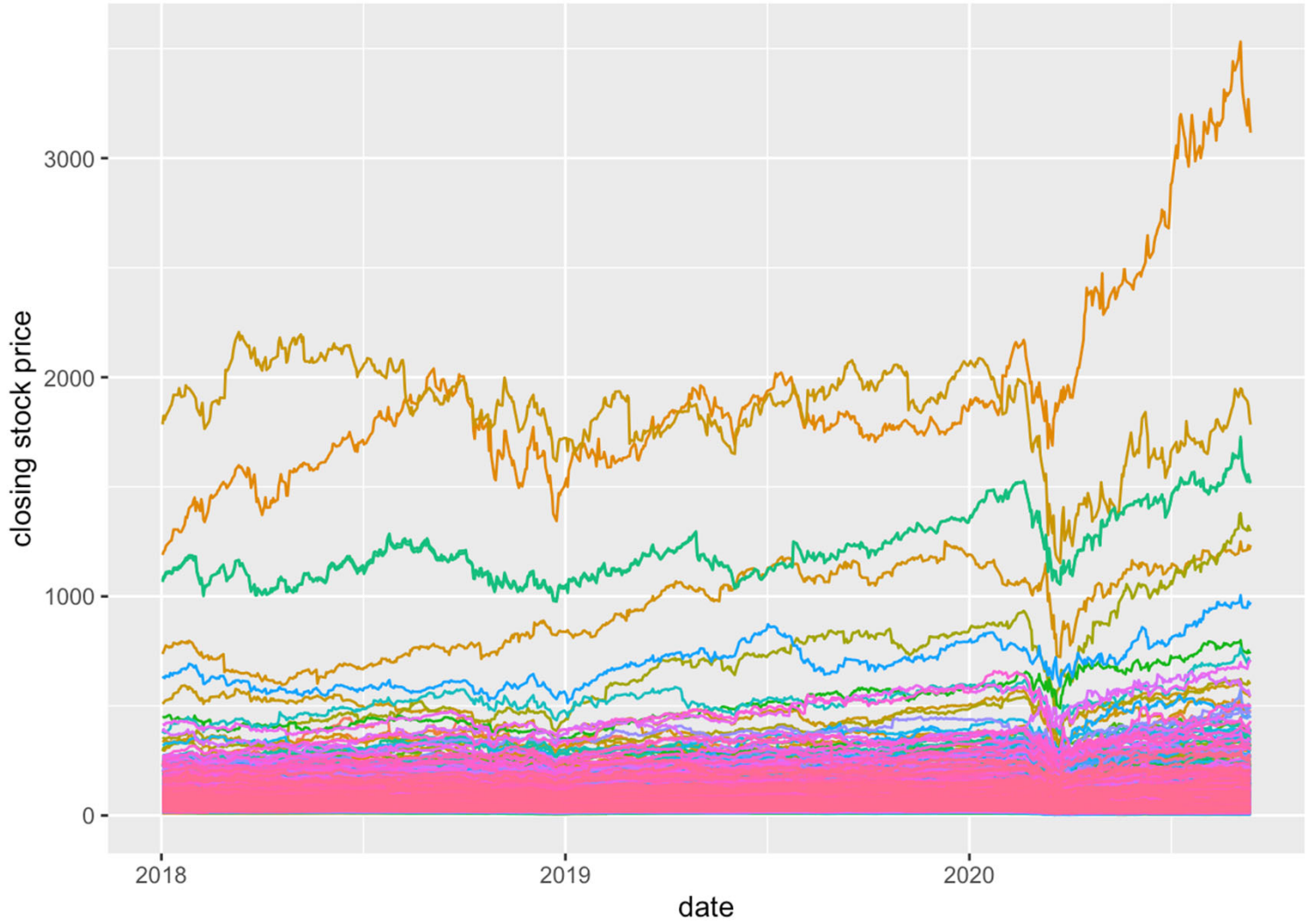
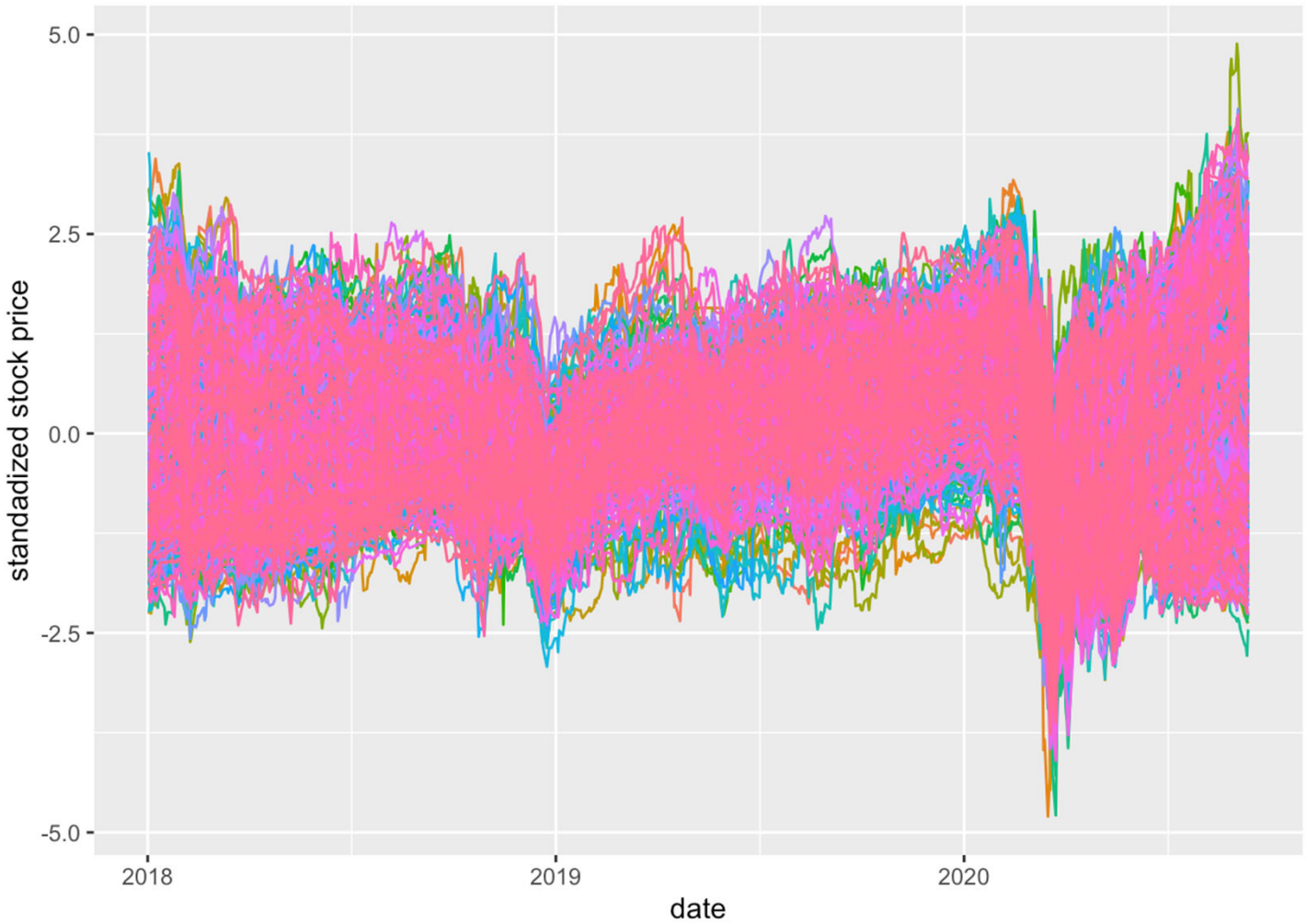
Clustering is subjective!!!

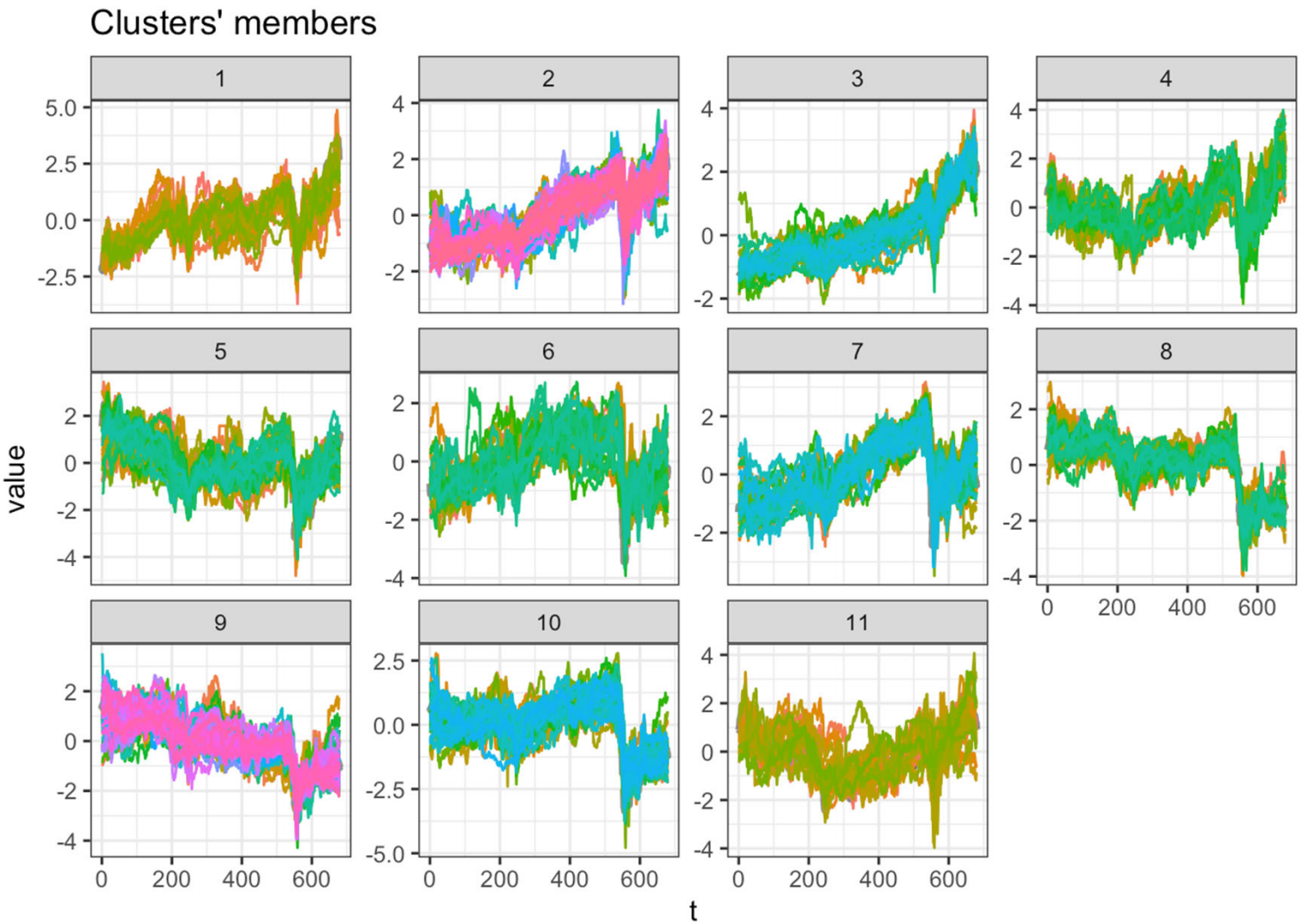
Motivation – Example Stock Prices (S&P 500)



Motivation – Example Stock Prices (S&P 500)



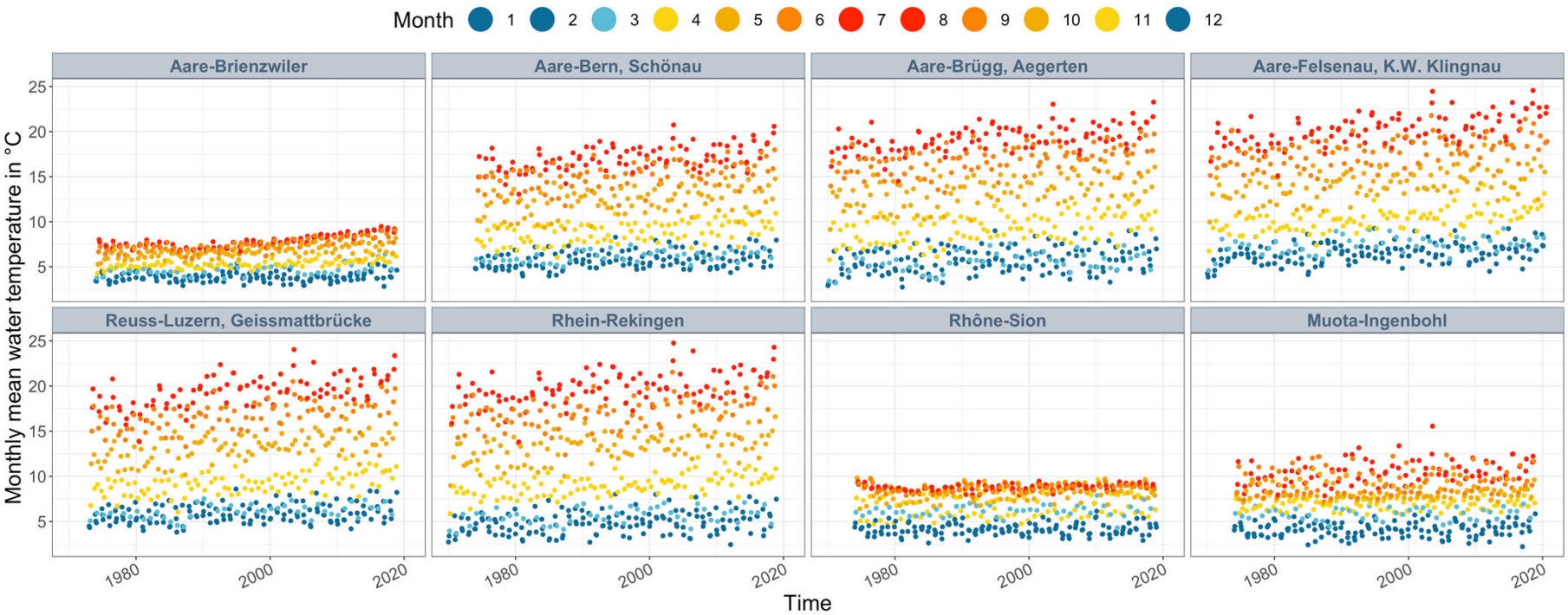
Motivation – Example Stock Prices (S&P 500)



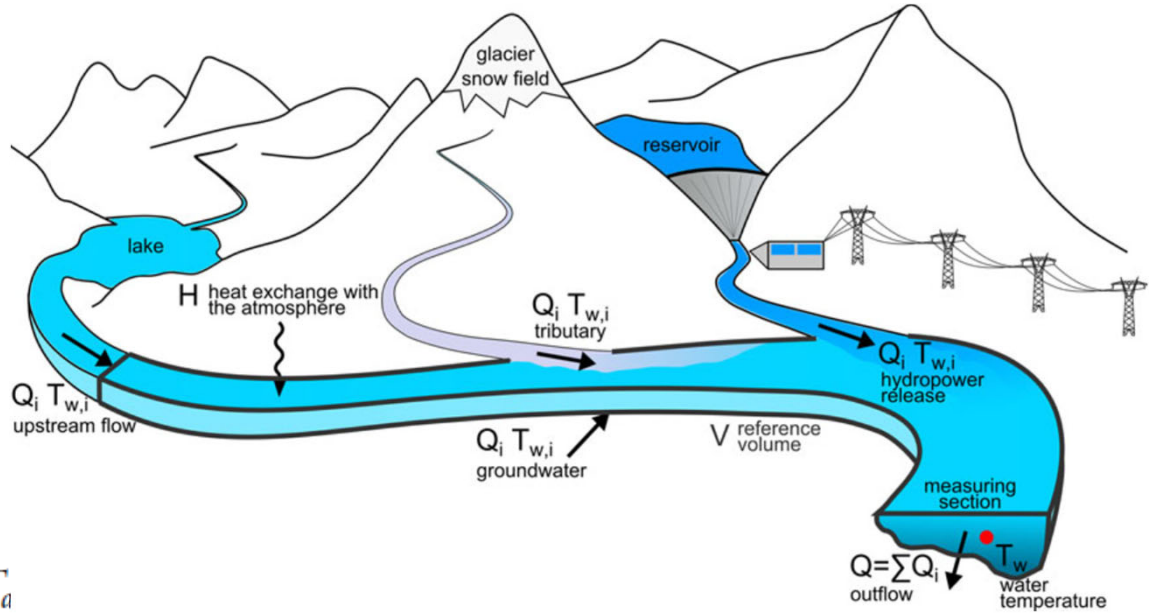
Motivation – The Challenge for FOEN

Rising water temperatures [1-7]

Federal policies



air2stream [6-11]



$$\frac{\mathrm{d}T_w}{\mathrm{d}t} = \frac{1}{\delta}\left\{ a_1 + a_2 T_a + \theta\left[a_5 + a_6 \cos\left(2\pi\left(\frac{t}{t_y} - a_7\right)\right) - a_8 T_w \right]\right\},$$

$$\delta = \theta^{a_4}, \quad \theta = Q/\overline{Q}$$

Input

River temperatur T_w

Air temperature T_a

Discharge Q

Estimate

Heat flux

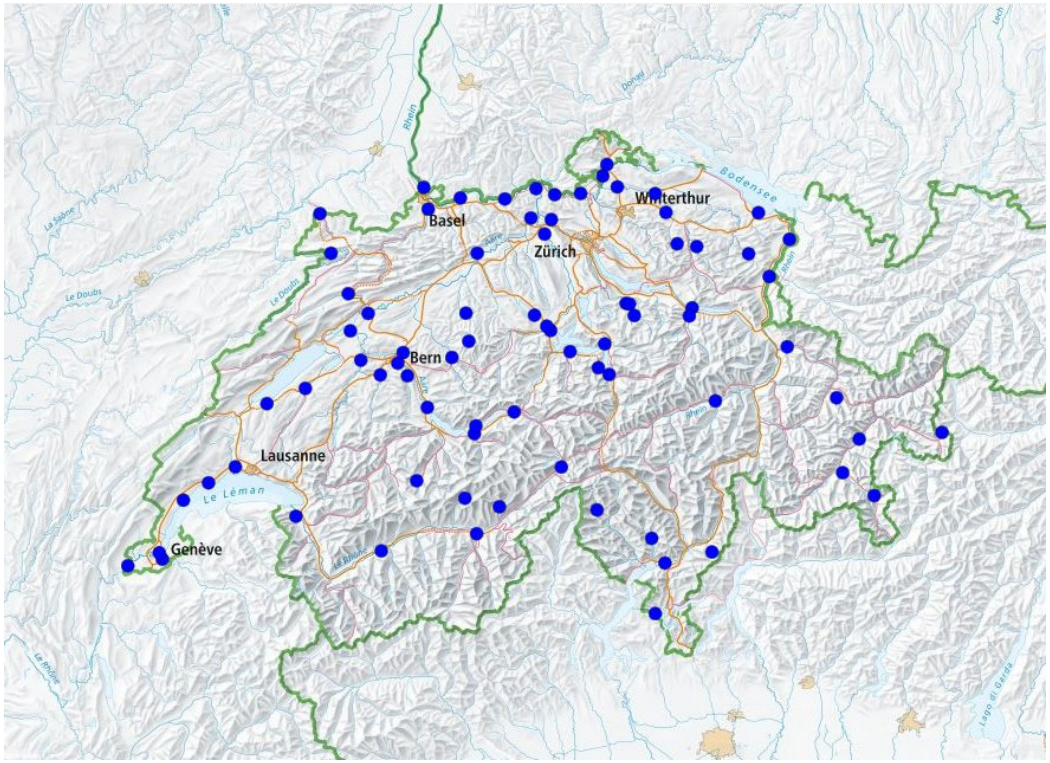
Seasonality

Variation Discharge θ

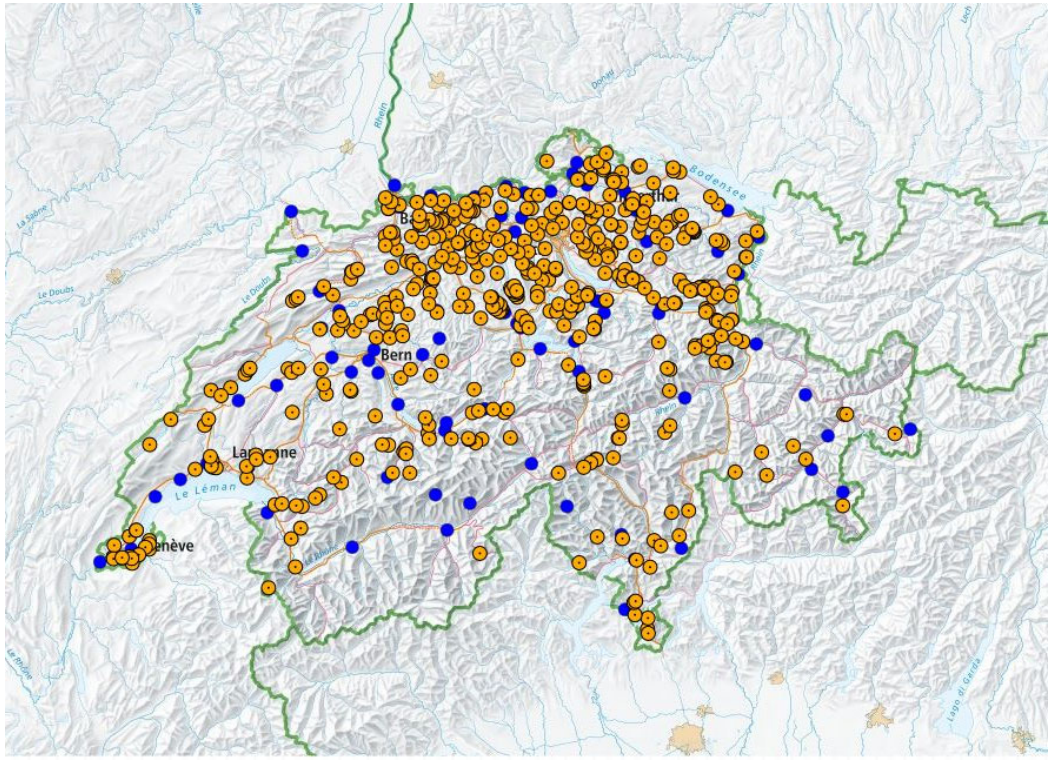
River depth δ

Motivation - The Challenge for FOEN

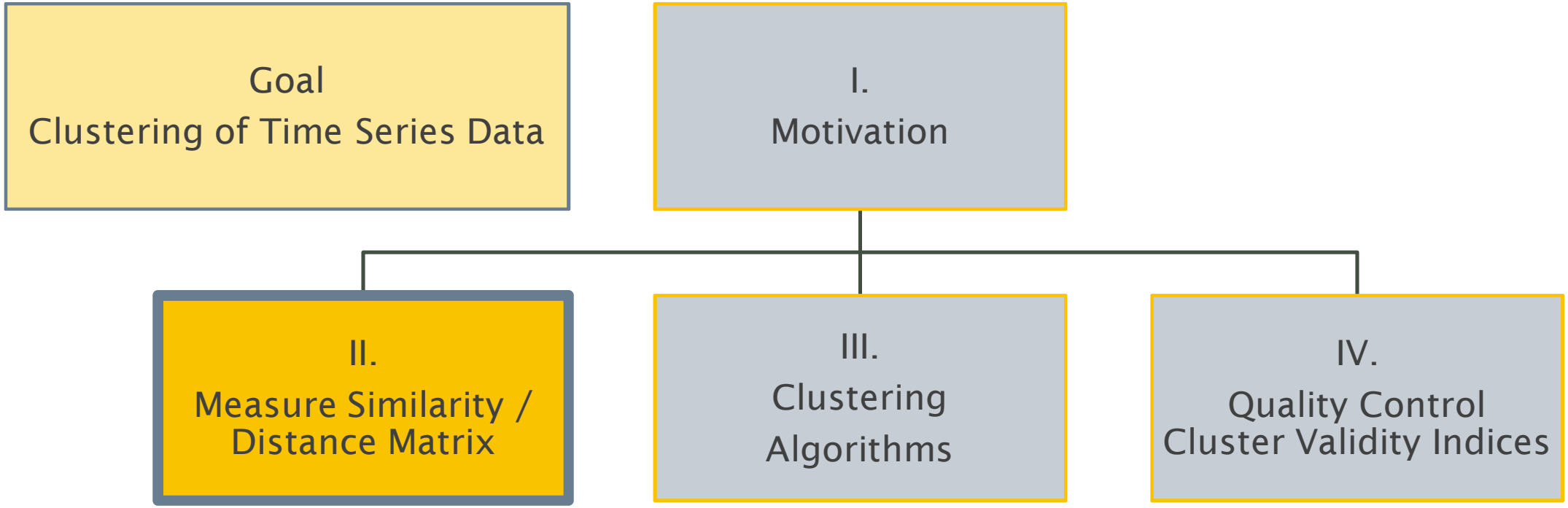
There are around 80 active FOEN measurement stations (blue).



Around 800 cantonal measurement stations exist (orange).



Structure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Measure Similarity /
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Distance Measures

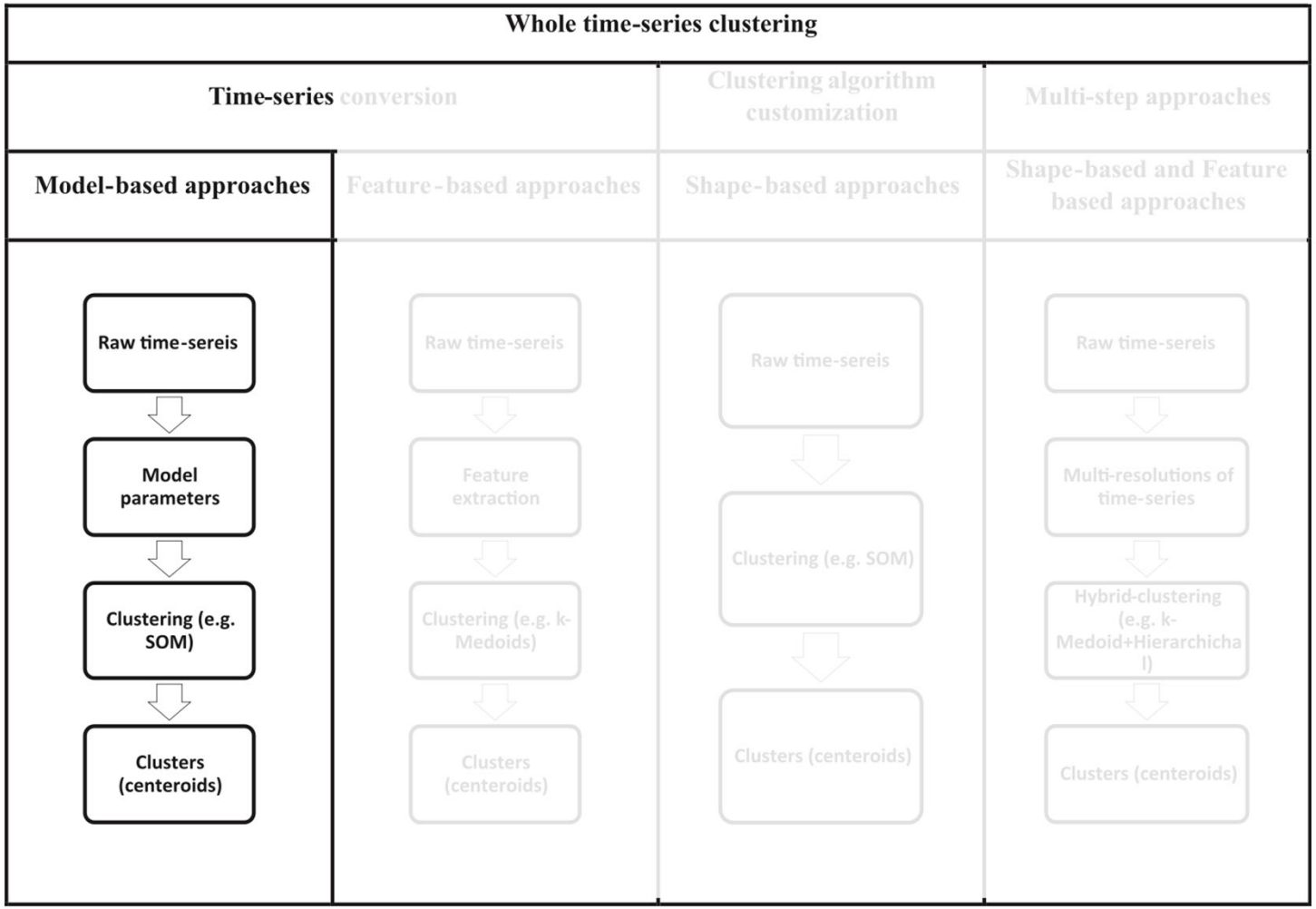


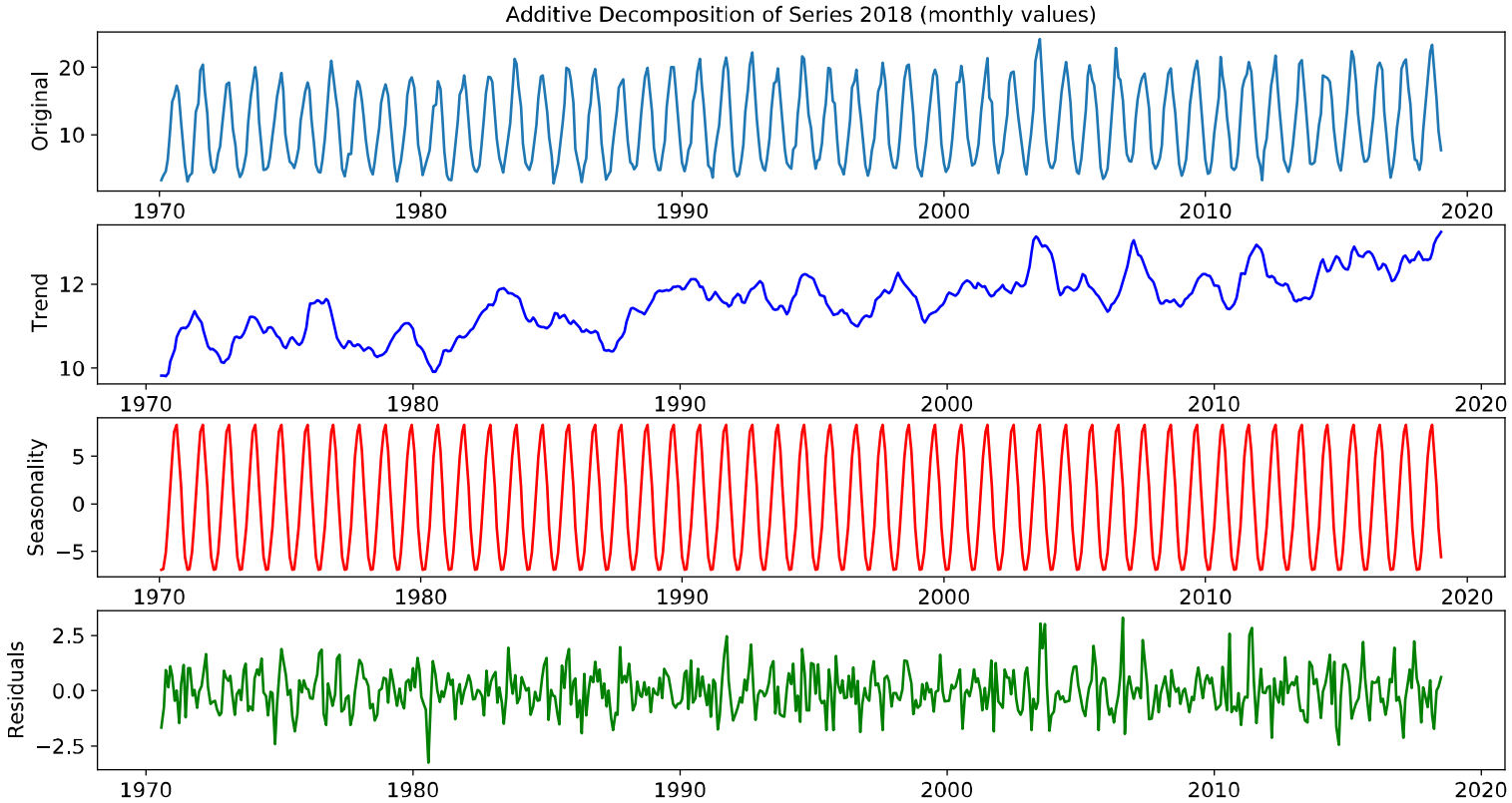
Fig. 2. The time-series clustering approaches.

Source [12]

Model-based Approach - Patterns in Time Series

Trend, Seasonality, Cycles

Additive Decomposition (O = T + S + R)



Distance Measures

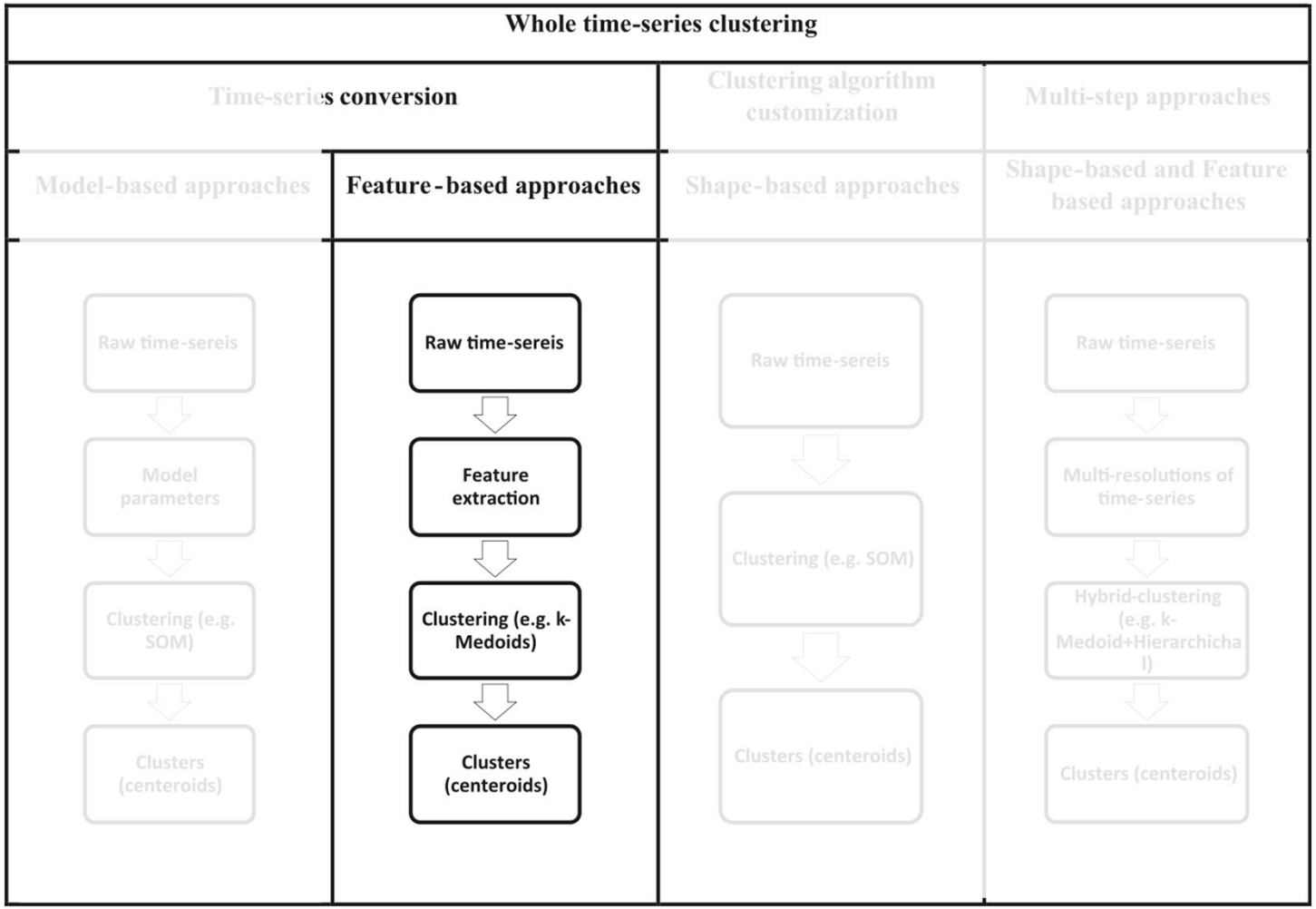
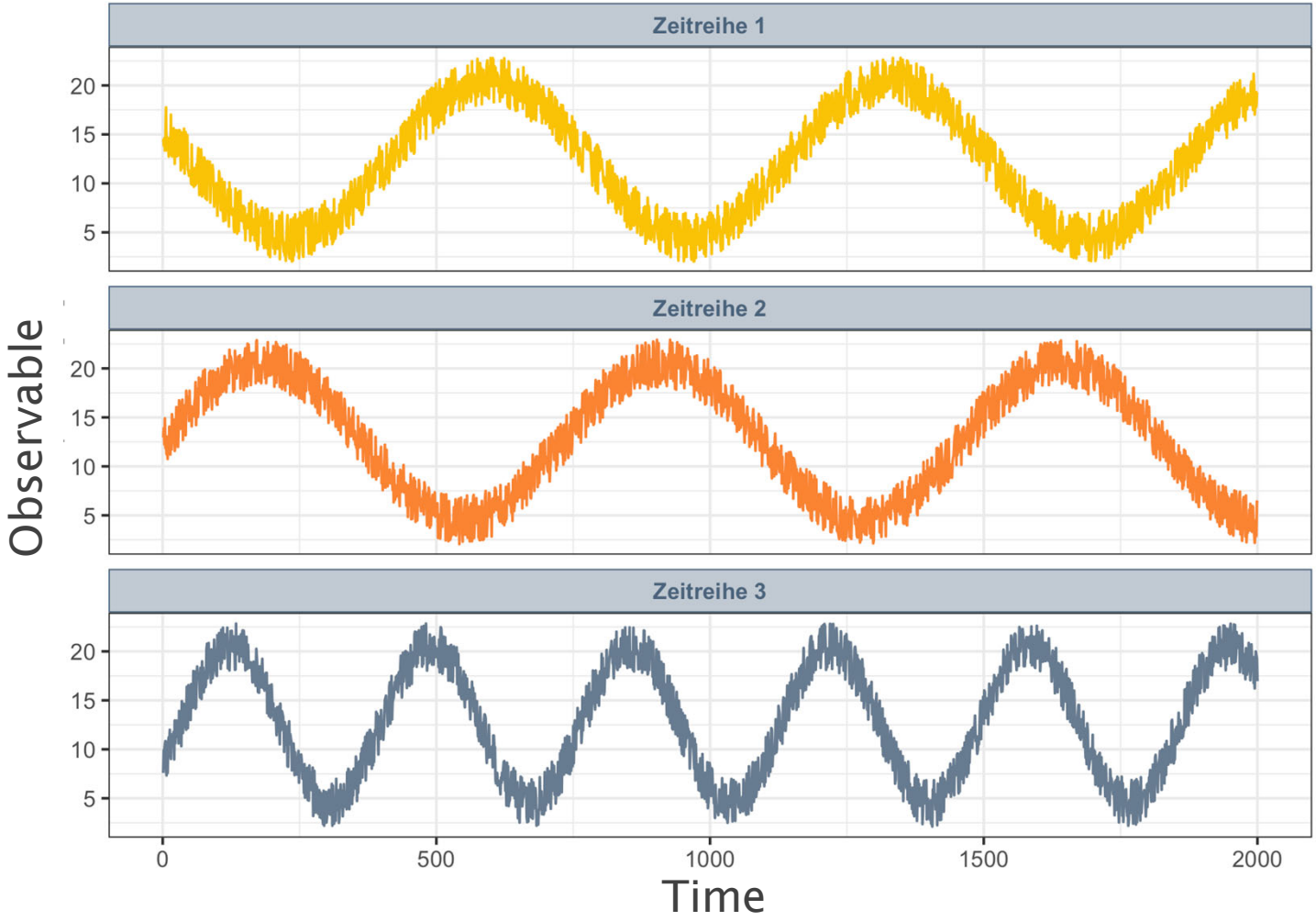


Fig. 2. The time-series clustering approaches.

Source [12]

Feature-based Approach – Simple Example



Nr.	Mean	Max.	Min.	Std.
1.	13.0	22.9	2	5.8
2.	11.8	22.8	2	5.7
3.	12.9	22.8	2.1	5.9

Distance Measures

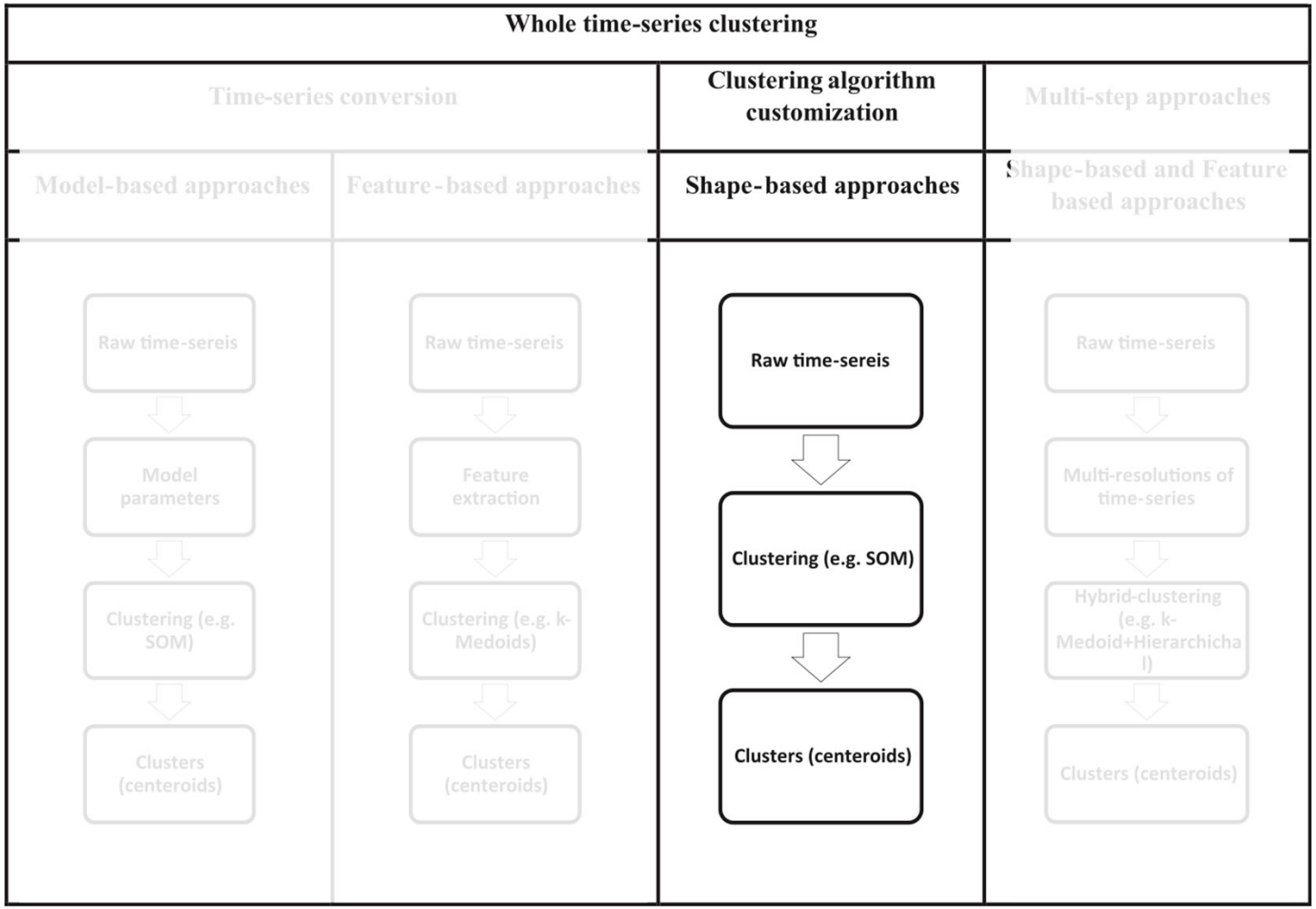


Fig. 2. The time-series clustering approaches.

Source [12]

Example

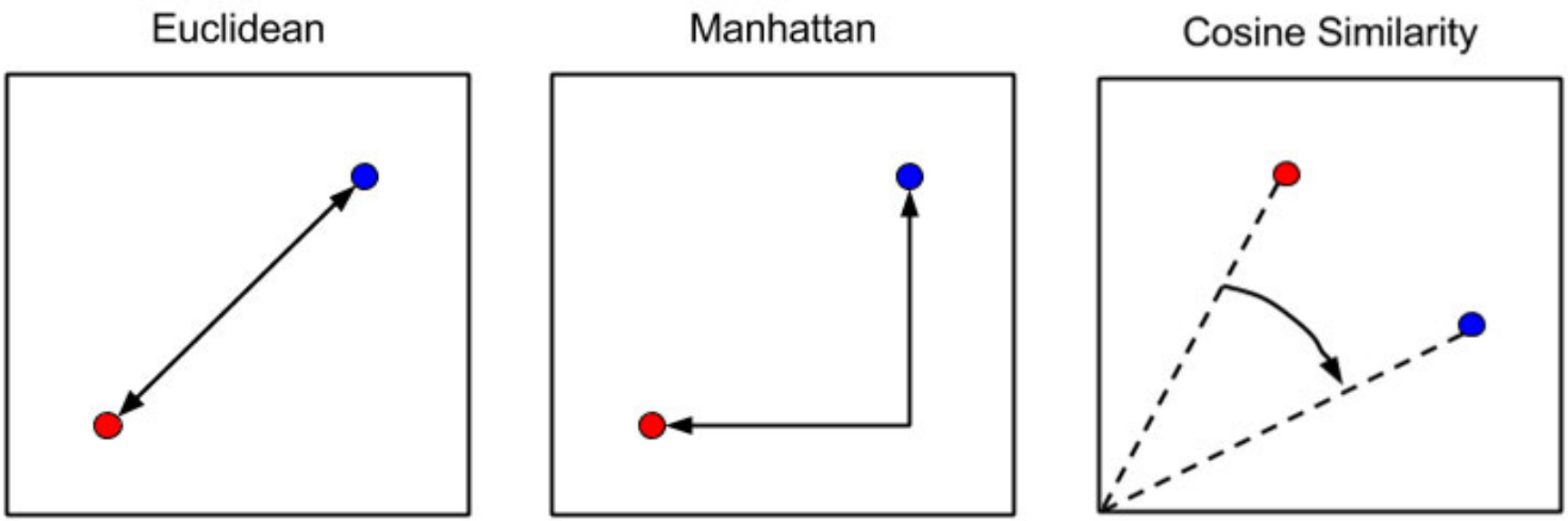
BAD IDEA
2 h 11 min
154 km
1 h 51 min
123 km
2 h 24 min
139 km
Zürich
Bern
Baden
Aarau
Olten
Oftringen
Solothurn
Langenthal
Burgdorf
Willisau
Lucerne
Zug
Baar
Kloten
Winterth
Schaffhausen
Waldshut-Tiengen
Jurapark Aargau
Regionaler Naturpark Thal
Delémont
Lyss
Langnau im Emmental
Belp
Naturpark Gantrisch
Stans
Schwyz
Altdorf
Engelberg
Stäfa

Distance Measures

In mathematics, a metric or distance function defines a distance between each pair of point elements of a set. Let a, b and c be arbitrary point elements of a set X, then the distance d satisfies the following three axioms:

1. $d(a,b) = 0 \iff a = b\,,$ *identity of indiscernibles*
2. $d(a,b) = d(b,a)\,,$ *symmetry*
3. $d(a,b) \leq d(a,c) + d(c,b)\,.$ *triangle inequality*

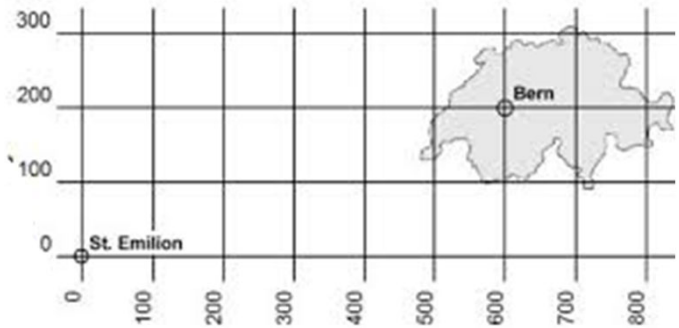
Distance Measures



Source [13]

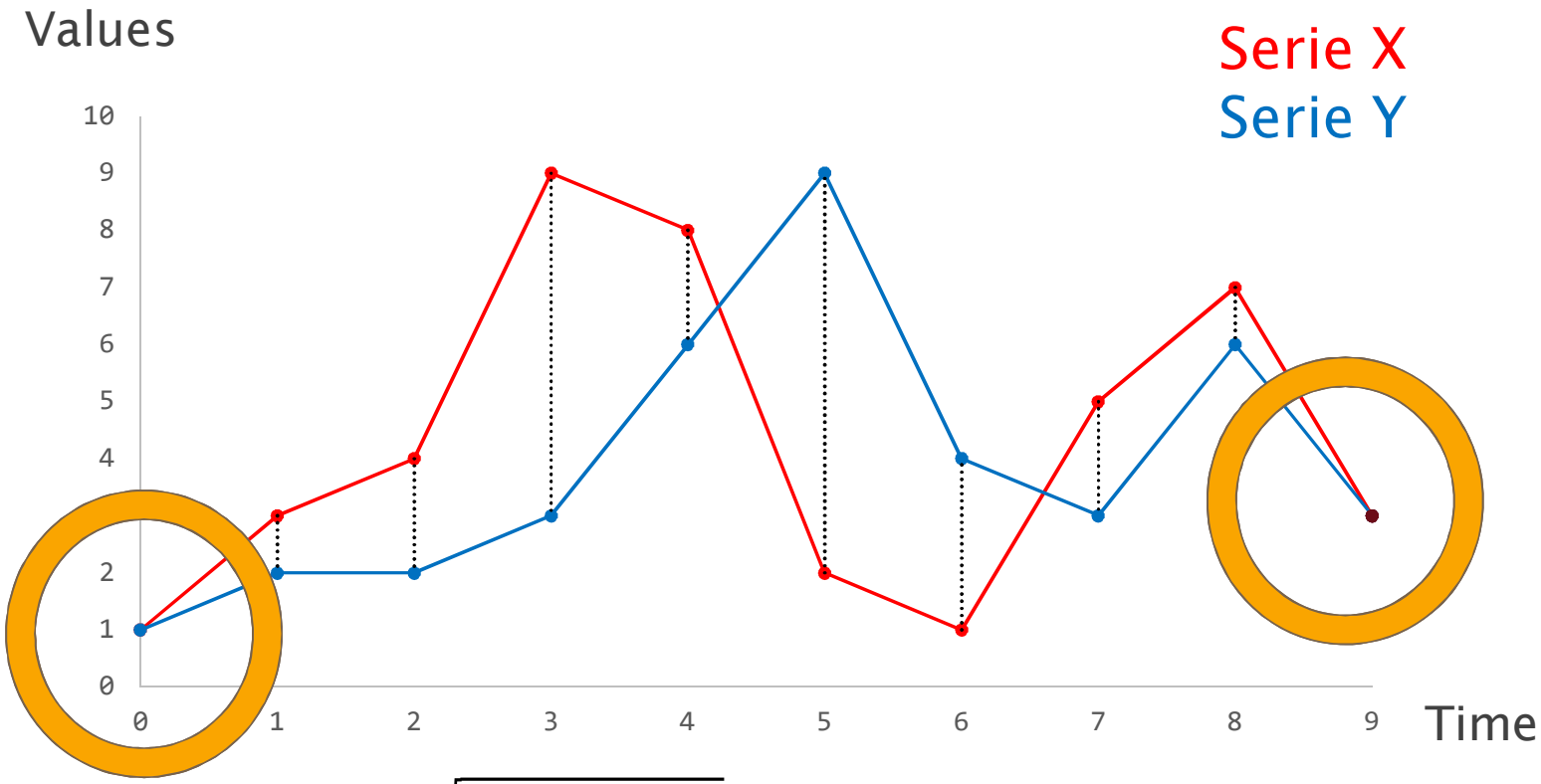
Euclidean Distance

In the old Swiss coordinate system (LV03) Bern-Siedlerstrasse has the coordinates (600′000,200′000) and Zürich-Seebach has the coordinates (684′592,252′857). Calculate the Euclidean distance between these cities.



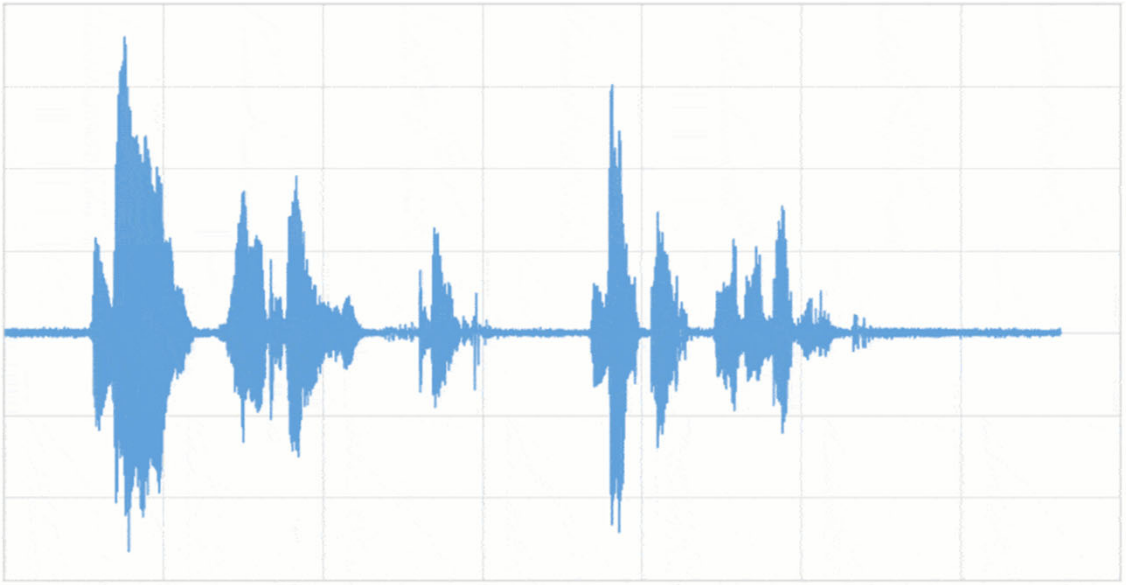
$$
\begin{aligned}
d(\text{Bern}, \text{Zuerich}) &= \left((600'000 - 684'592)^2 + (200'000 - 252'857)^2\right)^{0.5} \\
&= \left((-84592)^2 + (-52857)^2\right)^{0.5} \\
&= \sqrt{9949668913} \approx 99748.03
\end{aligned}
$$

Euclidean Distance (Pairwise Distance)

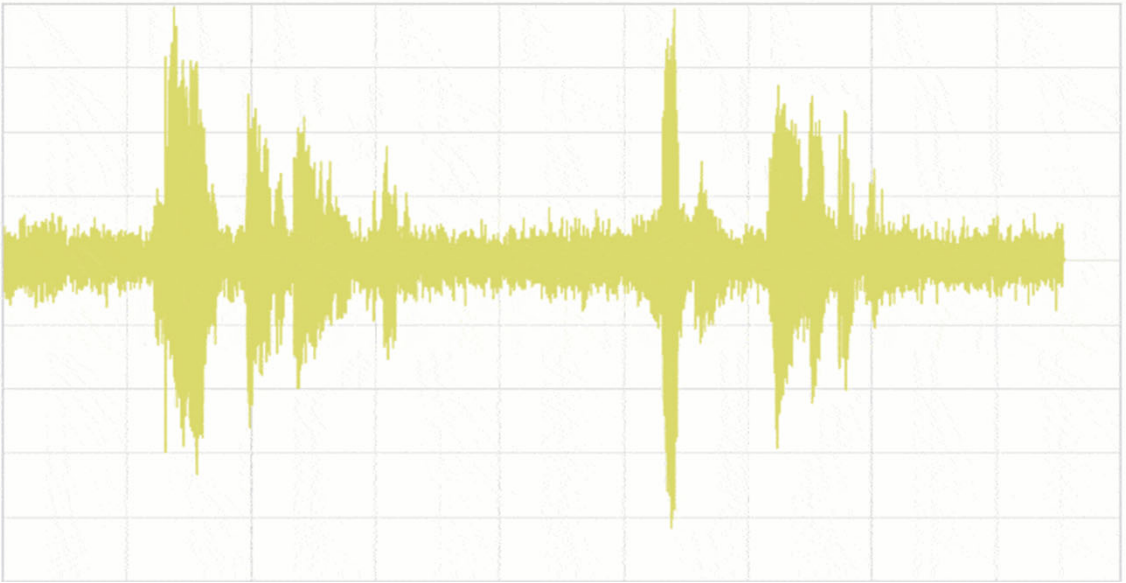


$$d_{Euclidean}(X,Y) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2} = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \ldots + (x_n - y_n)^2}$$

Example



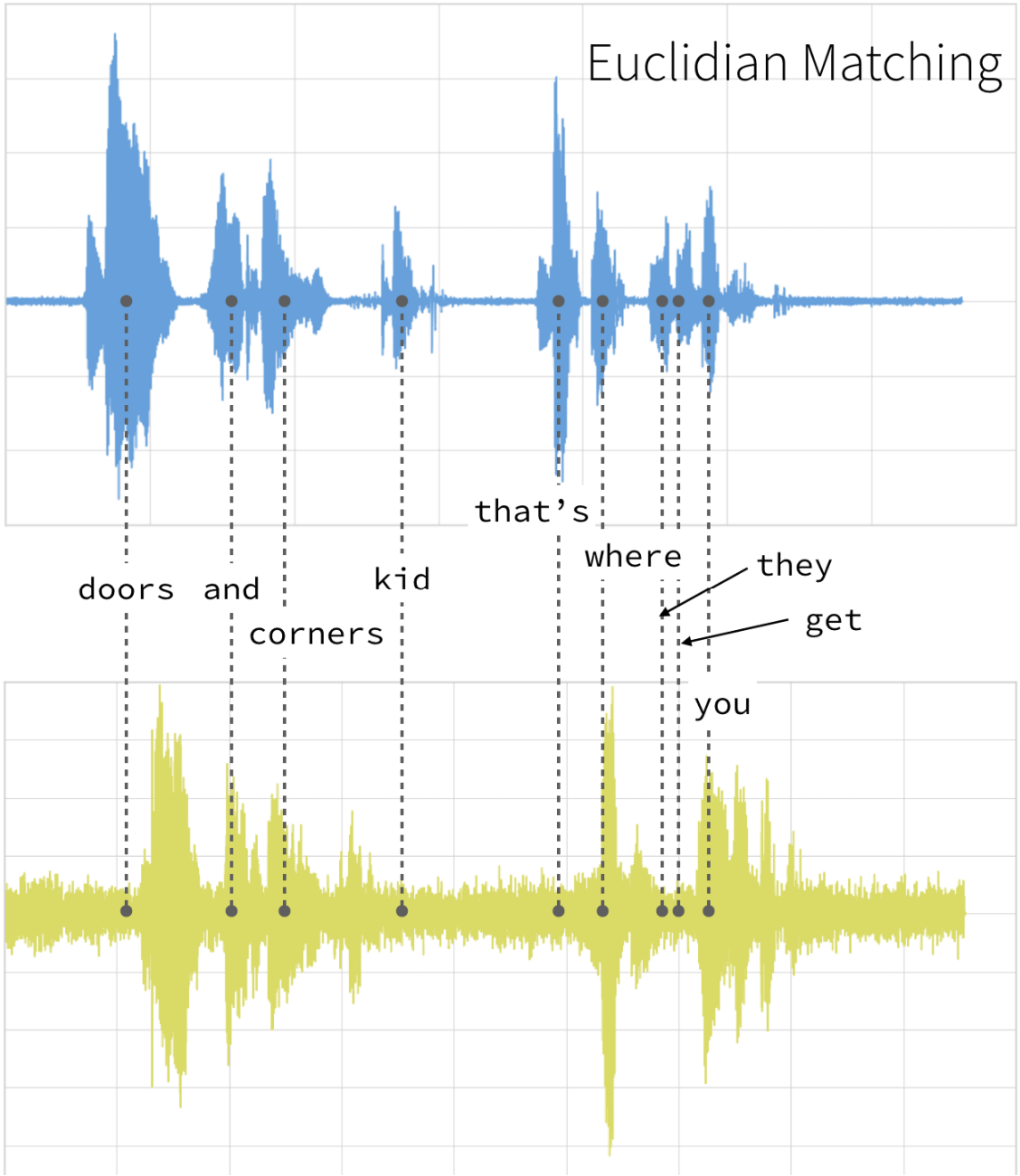
Doors and corners, kid. That's where they get you.



Source [14]

Example

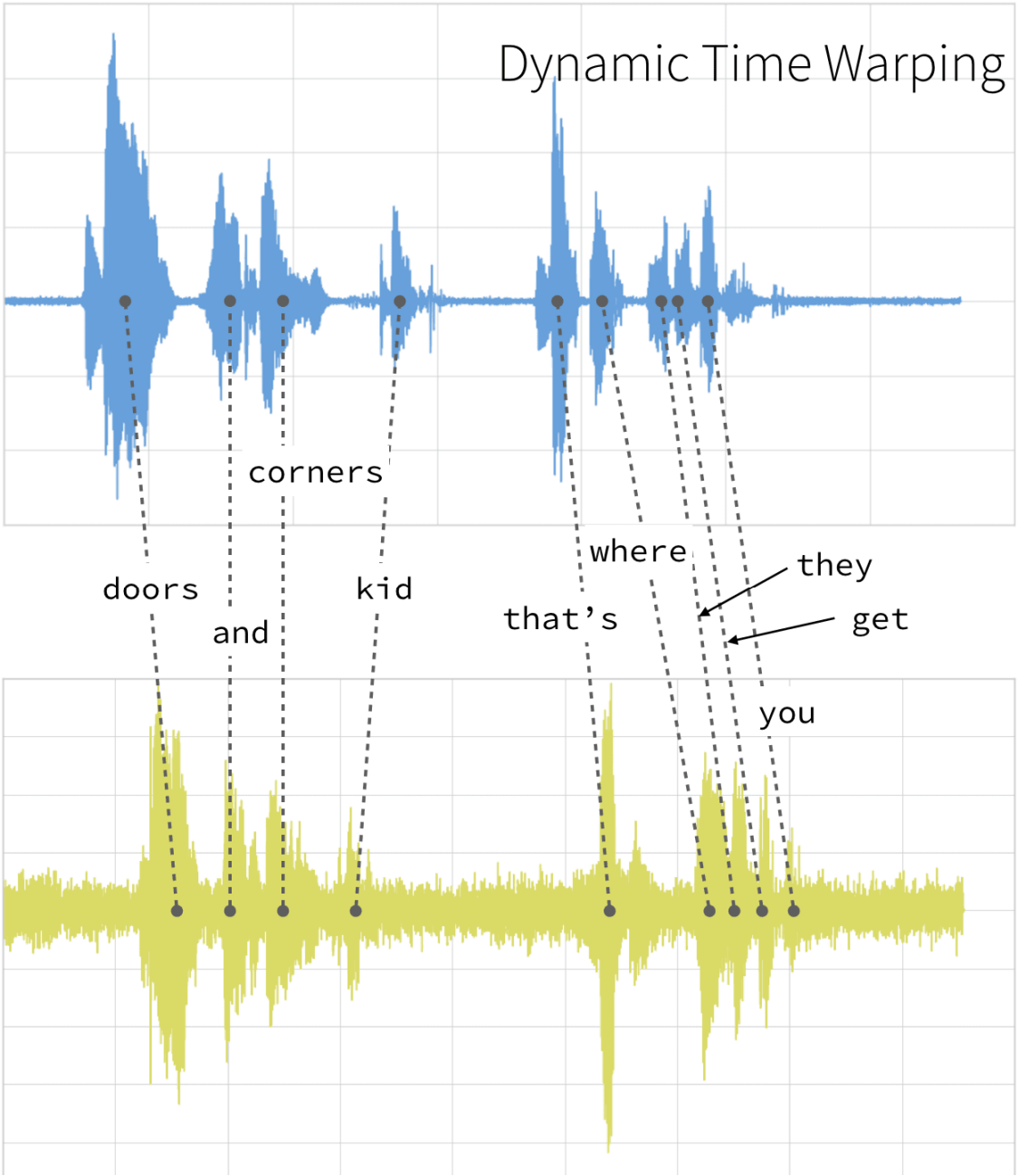
Disadvantage of Pairwise Distance Measures



Source [14]

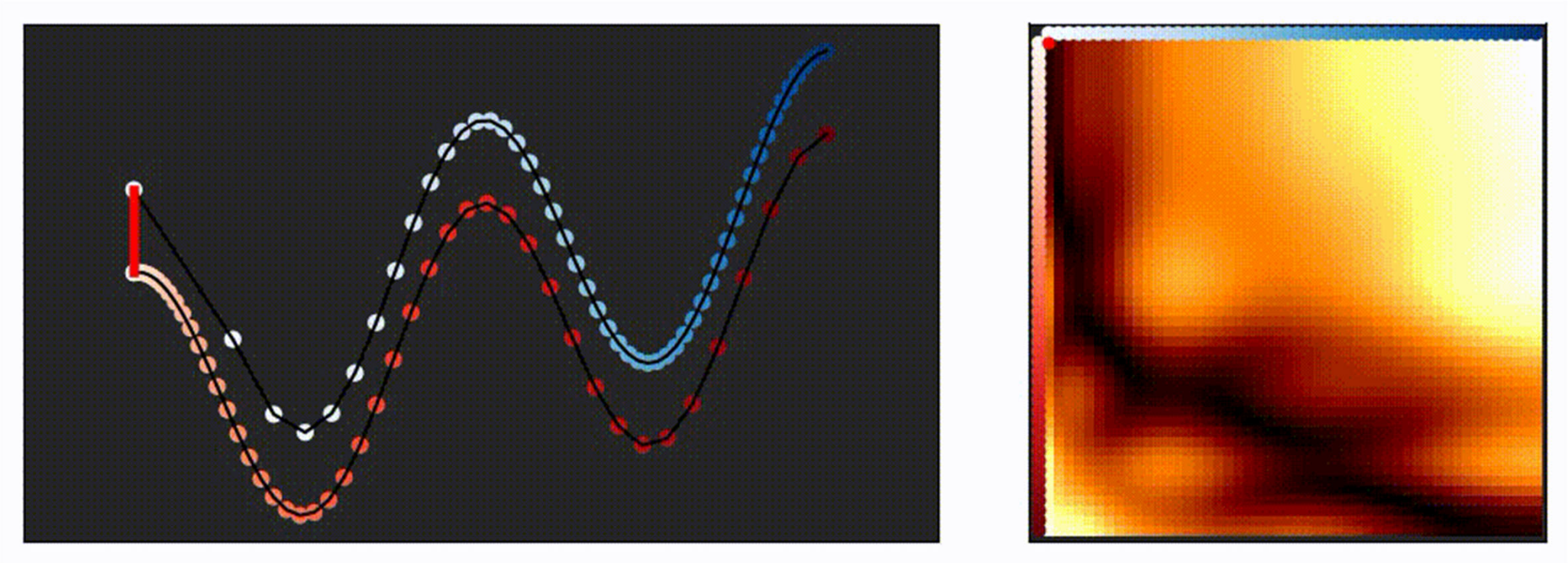
Example

This is what you hope to achieve!



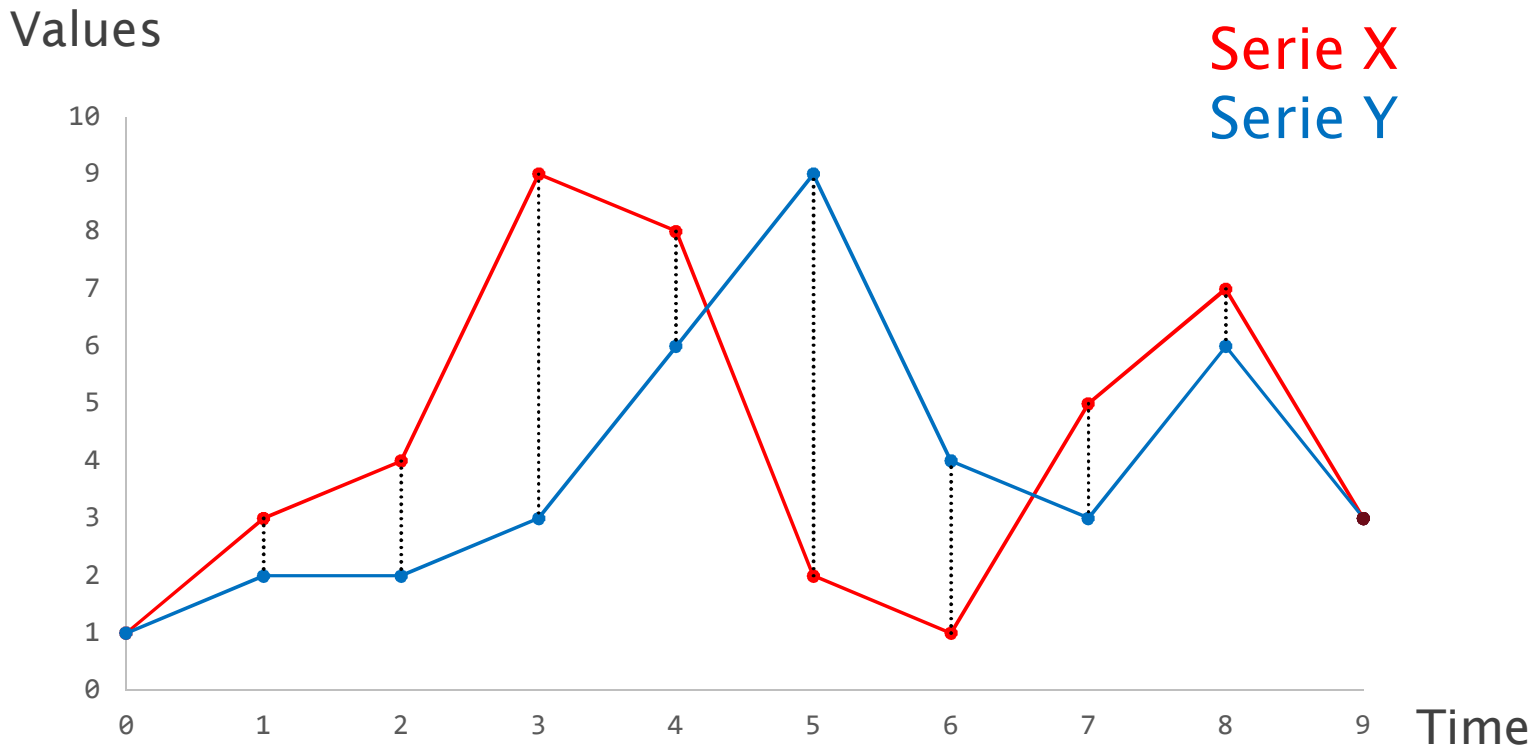
Source [14]

Dynamic Time Warping DTW



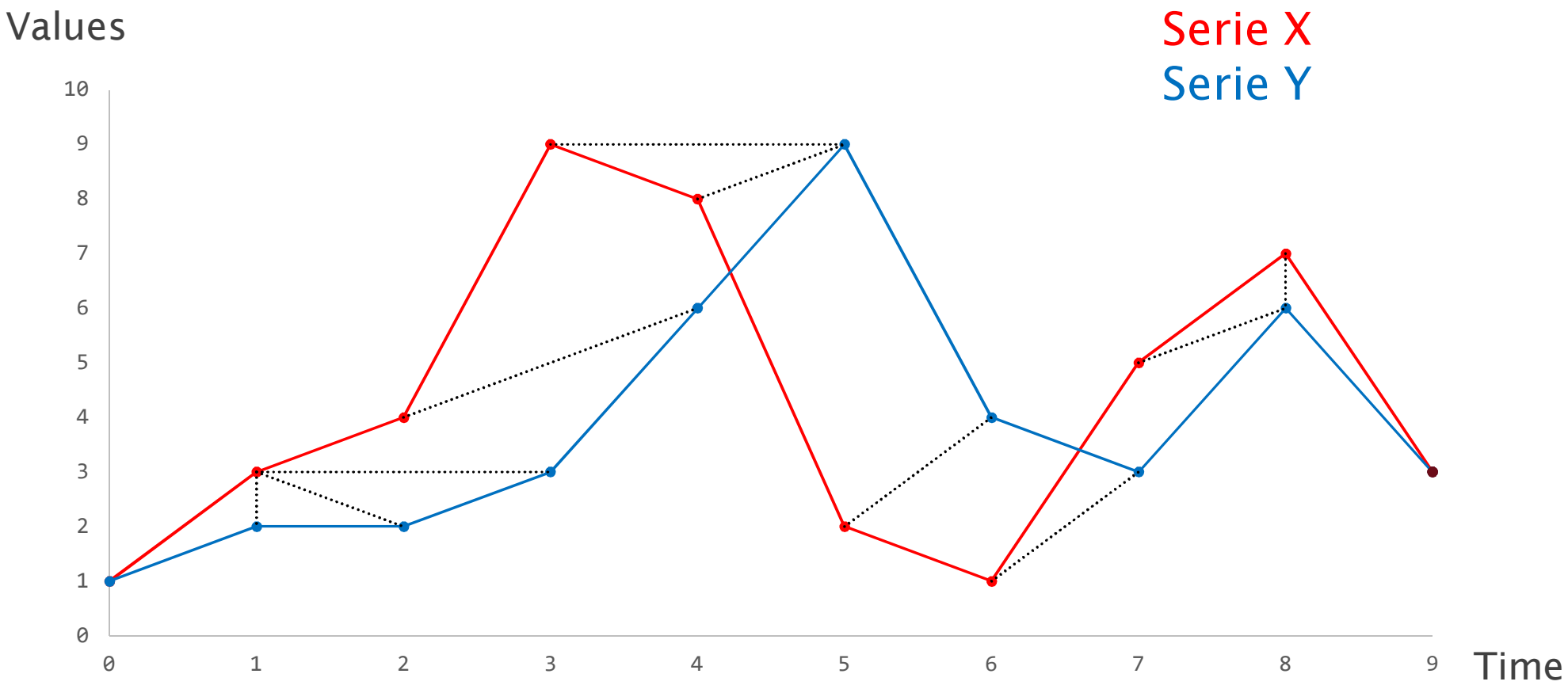
Source [15]

Euclidean Distance (Pairwise Distance)



$$d_{Euclidean}(X,Y) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2} = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \ldots + (x_n - y_n)^2}$$

Example - Dynamic Time Warping DTW

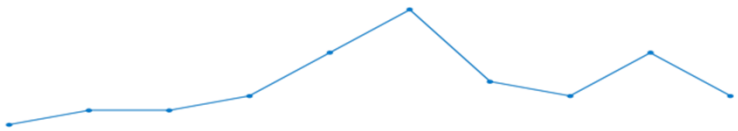


Example - Dynamic Time Warping DTW

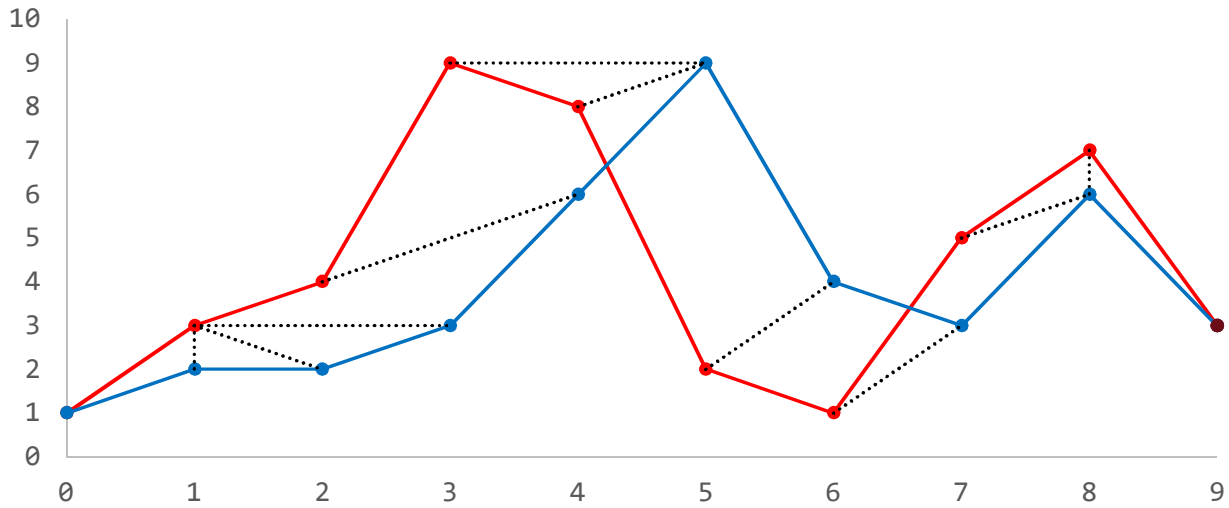
Serie A

9	3									
8	7									
7	5									
6	1									
5	2									
4	8									
3	9									
2	4									
1	3									
0	1									
i		1	2	2	3	6	9	4	3	6	3
	j	0	1	2	3	4	5	6	7	8	9

Serie B



Manhattan Distance



Example - Dynamic Time Warping DTW

Manhattan Distance

Serie A	9	3									
	8	7									
	7	5									
	6	1									
	5	2									
	4	8									
	3	9									
	2	4									
	1	3									
	0	1	0								
	i		1	2	2	3	6	9	4	3	6	3
		j	0	1	2	3	4	5	6	7	8	9
			Serie B									

$$|1 - 1| = 0$$

Example - Dynamic Time Warping DTW

Serie A

9	3									
8	7									
7	5									
6	1									
5	2									
4	8									
3	9									
2	4									
1	3	2								
0	1	0								
i		1	2	2	3	6	9	4	3	6	3
	j	0	1	2	3	4	5	6	7	8	9

$$|3-1|+0=2$$

Serie B

Example - Dynamic Time Warping DTW

Serie A

9	3									
8	7									
7	5									
6	1									
5	2									
4	8									
3	9									
2	4	5								
1	3	2								
0	1	0								
i		1	2	2	3	6	9	4	3	6	3
	j	0	1	2	3	4	5	6	7	8	9

$$|4 - 1| + 2 = 5$$

Serie B

Example - Dynamic Time Warping DTW

$$|1-2|+0=1$$

Serie A	9	3									
	8	7									
	7	5									
	6	1									
	5	2									
	4	8									
	3	9	13								
	2	4	5								
	1	3	2								
	0	1	0	1							
	i		1	2	2	3	6	9	4	3	6	3
		j	0	1	2	3	4	5	6	7	8	9
			Serie B									

Example - Dynamic Time Warping DTW

$$|1-2|+1=2$$

Serie A	9	3									
	8	7									
	7	5									
	6	1									
	5	2									
	4	8									
	3	9	13								
	2	4	5								
	1	3	2								
	0	1	0	1	2						
	i		1	2	2	3	6	9	4	3	6	3
		j	0	1	2	3	4	5	6	7	8	9
			Serie B									

Example - Dynamic Time Warping DTW

Serie A

9	3	33									
8	7	31									
7	5	25									
6	1	21									
5	2	21									
4	8	20									
3	9	13									
2	4	5									
1	3	2									
0	1	0	1	2	4	9	17	20	22	27	29
i		1	2	2	3	6	9	4	3	6	3
	j	0	1	2	3	4	5	6	7	8	9

Serie B

Example - Dynamic Time Warping DTW

Manhattan Distance

$$|3-2| + \min\{0, 1, 2\} = 1$$

	9	3	33								
	8	7	31								
	7	5	25								
	6	1	21								
	5	2	21								
	4	8	20								
	3	9	13								
	2	4	5								
	1	3	2	1							
	0	1	0	1	2	4	9	17	20	22	27	29
	i		1	2	2	3	6	9	4	3	6	3
		j	0	1	2	3	4	5	6	7	8	9

Serie A

Serie B

Example - Dynamic Time Warping DTW

$$|4-2| + \min\{2, 1, 5\} = 3$$

	9	3	33								
	8	7	31								
	7	5	25								
	6	1	21								
	5	2	21								
	4	8	20								
	3	9	13								
	2	4	5	3							
	1	3	2	1							
	0	1	0	1	2	4	9	17	20	22	27	29
	i		1	2	2	3	6	9	4	3	6	3
		j	0	1	2	3	4	5	6	7	8	9

Serie A

Serie B

Example - Dynamic Time Warping DTW

$$|3-2| + \min\{1, 2, 1\} = 2$$

	9	3	33								
	8	7	31								
	7	5	25								
	6	1	21								
	5	2	21								
	4	8	20								
	3	9	13								
	2	4	5	3							
	1	3	2	1	2						
	0	1	0	1	2	4	9	17	20	22	27	29
	i		1	2	2	3	6	9	4	3	6	3
		j	0	1	2	3	4	5	6	7	8	9

Serie A

Serie B

Example - Dynamic Time Warping DTW

$$|4-2| + \min\{1,2,3\} = 3$$

	9	3	33								
	8	7	31								
	7	5	25								
	6	1	21								
	5	2	21								
	4	8	20								
	3	9	13								
	2	4	5	3	3						
	1	3	2	1	2						
	0	1	0	1	2	4	9	17	20	22	27	29
	i		1	2	2	3	6	9	4	3	6	3
		j	0	1	2	3	4	5	6	7	8	9

Serie A

Serie B

Example - Dynamic Time Warping DTW

9	3	33	26	26	23	22	25	15	14	14	11
8	7	31	25	25	23	19	20	14	15	11	14
7	5	25	20	20	19	18	21	11	11	10	12
6	1	21	17	17	17	17	20	10	9	13	14
5	2	21	16	16	15	12	12	7	8	12	13
4	8	20	16	16	14	8	5	8	13	15	18
3	9	13	10	10	9	6	4	9	15	13	18
2	4	5	3	3	3	4	9	9	10	12	13
1	3	2	1	2	2	5	11	12	12	15	15
0	1	0	1	2	4	9	17	20	22	27	29
i		1	2	2	3	6	9	4	3	6	3
	j	0	1	2	3	4	5	6	7	8	9

Serie A

Serie B

Example - Dynamic Time Warping DTW

9	3	33	26	26	23	22	25	15	14	14	11
8	7	31	25	25	23	19	20	14	15	11	14
7	5	25	20	20	19	18	21	11	11	10	12
6	1	21	17	17	17	17	20	10	9	13	14
5	2	21	16	16	15	12	12	7	8	12	13
4	8	20	16	16	14	8	5	8	13	15	18
3	9	13	10	10	9	6	4	9	15	13	18
2	4	5	3	3	3	4	9	9	10	12	13
1	3	2	1	2	2	5	11	12	12	15	15
0	1	0	1	2	4	9	17	20	22	27	29
i		1	2	2	3	6	9	4	3	6	3
	j	0	1	2	3	4	5	6	7	8	9

Serie A

Serie B

Example - Dynamic Time Warping DTW

9	3	33	26	26	23	22	25	15	14	14	11
8	7	31	25	25	23	19	20	14	15	11	14
7	5	25	20	20	19	18	21	11	11	10	12
6	1	21	17	17	17	17	20	10	9	13	14
5	2	21	16	16	15	12	12	7	8	12	13
4	8	20	16	16	14	8	5	8	13	15	18
3	9	13	10	10	9	6	4	9	15	13	18
2	4	5	3	3	3	4	9	9	10	12	13
1	3	2	1	2	2	5	11	12	12	15	15
0	1	0	1	2	4	9	17	20	22	27	29
i		1	2	2	3	6	9	4	3	6	3
	j	0	1	2	3	4	5	6	7	8	9

Serie A

Serie B

Example - Dynamic Time Warping DTW

9	3	33	26	26	23	22	25	15	14	14	11
8	7	31	25	25	23	19	20	14	15	11	14
7	5	25	20	20	19	18	21	11	11	10	12
6	1	21	17	17	17	17	20	10	9	13	14
5	2	21	16	16	15	12	12	7	8	12	13
4	8	20	16	16	14	8	5	8	13	15	18
3	9	13	10	10	9	6	4	9	15	13	18
2	4	5	3	3	3	4	9	9	10	12	13
1	3	2	1	2	2	5	11	12	12	15	15
0	1	0	1	2	4	9	17	20	22	27	29
i		1	2	2	3	6	9	4	3	6	3
	j	0	1	2	3	4	5	6	7	8	9

Serie A

Serie B

$$\min(14, 11, 14) = 11$$

Example - Dynamic Time Warping DTW

9	3	33	26	26	23	22	25	15	14	14	11
8	7	31	25	25	23	19	20	14	15	11	14
7	5	25	20	20	19	18	21	11	11	10	12
6	1	21	17	17	17	17	20	10	9	13	14
5	2	21	16	16	15	12	12	7	8	12	13
4	8	20	16	16	14	8	5	8	13	15	18
3	9	13	10	10	9	6	4	9	15	13	18
2	4	5	3	3	3	4	9	9	10	12	13
1	3	2	1	2	2	5	11	12	12	15	15
0	1	0	1	2	4	9	17	20	22	27	29
i		1	2	2	3	6	9	4	3	6	3
	j	0	1	2	3	4	5	6	7	8	9

Serie A

Serie B

Example - Dynamic Time Warping DTW

9	3	33	26	26	23	22	25	15	14	14	11
8	7	31	25	25	23	19	20	14	15	11	14
7	5	25	20	20	19	18	21	11	11	10	12
6	1	21	17	17	17	17	20	10	9	13	14
5	2	21	16	16	15	12	12	7	8	12	13
4	8	20	16	16	14	8	5	8	13	15	18
3	9	13	10	10	9	6	4	9	15	13	18
2	4	5	3	3	3	4	9	9	10	12	13
1	3	2	1	2	2	5	11	12	12	15	15
0	1	0	1	2	4	9	17	20	22	27	29
i		1	2	2	3	6	9	4	3	6	3
	j	0	1	2	3	4	5	6	7	8	9

Serie A

Serie B

$$\min(15, 11, 10) = 10$$

Example - Dynamic Time Warping DTW

9	3	33	26	26	23	22	25	15	14	14	11
8	7	31	25	25	23	19	20	14	15	11	14
7	5	25	20	20	19	18	21	11	11	10	12
6	1	21	17	17	17	17	20	10	9	13	14
5	2	21	16	16	15	12	12	7	8	12	13
4	8	20	16	16	14	8	5	8	13	15	18
3	9	13	10	10	9	6	4	9	15	13	18
2	4	5	3	3	3	4	9	9	10	12	13
1	3	2	1	2	2	5	11	12	12	15	15
0	1	0	1	2	4	9	17	20	22	27	29
i		1	2	2	3	6	9	4	3	6	3
	j	0	1	2	3	4	5	6	7	8	9

Serie A

Serie B

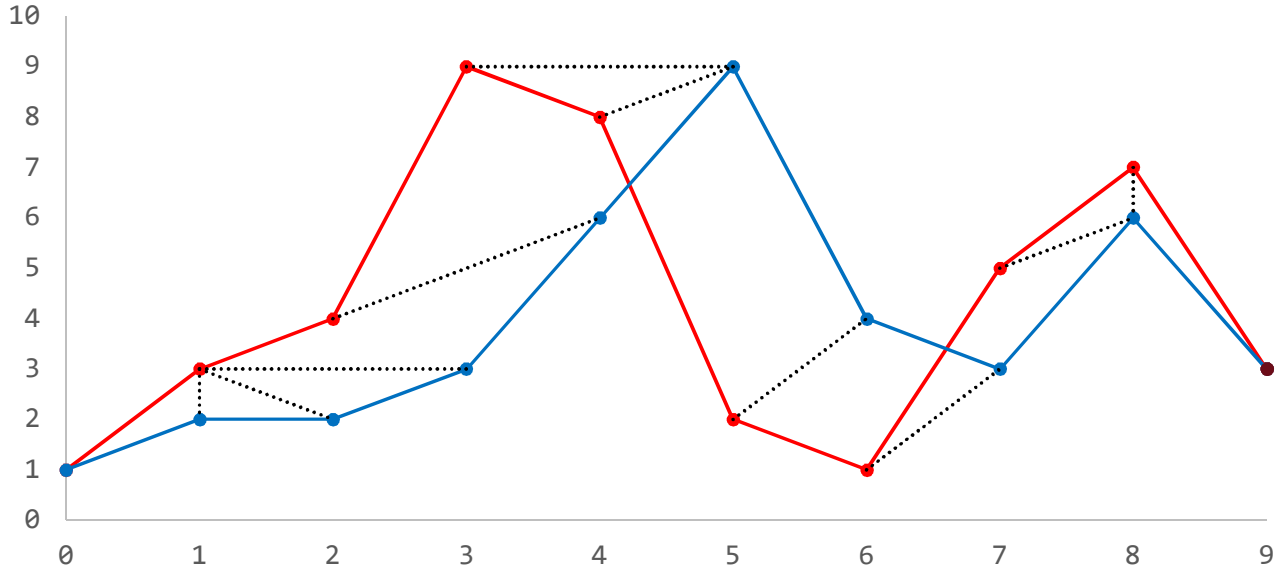
Minimum Spanning Tree

Warping Path

Serie A

4	8	20	16	16	14	8	5	8
3	9	13	10	10	9	6	4	9
2	4	5	3	3	3	4	9	9
1	3	2	1	2	2	5	11	12
0	1	0	1	2	4	9	17	20
i		1	2	2	3	6	9	4
	j	0	1	2	3	4	5	6

Serie B



Comment 1: Constraints – Window Size and Shape

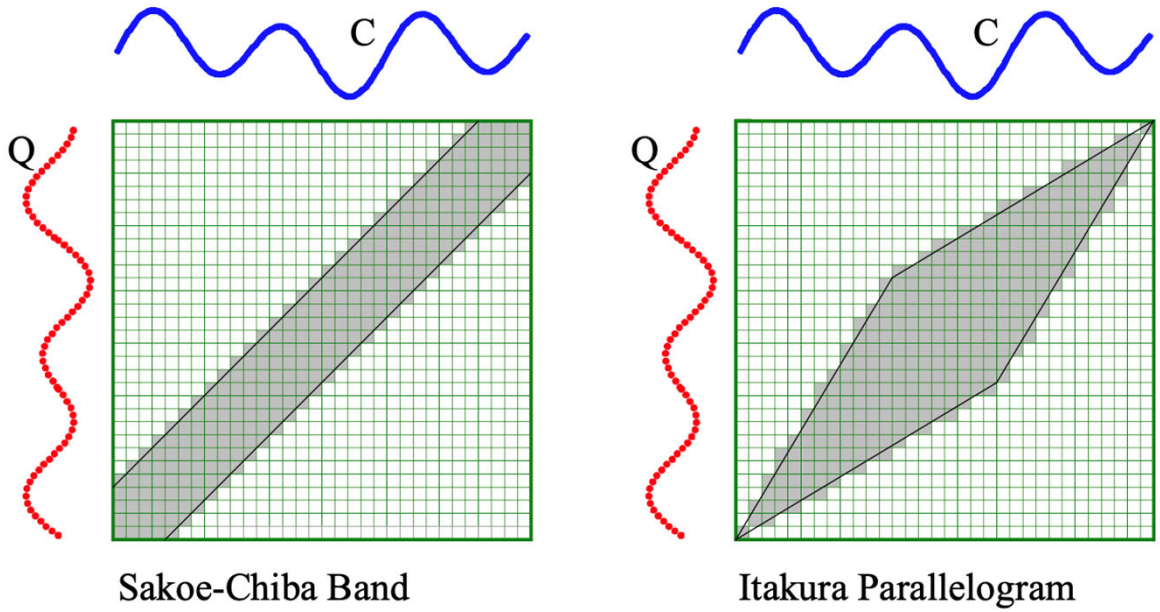
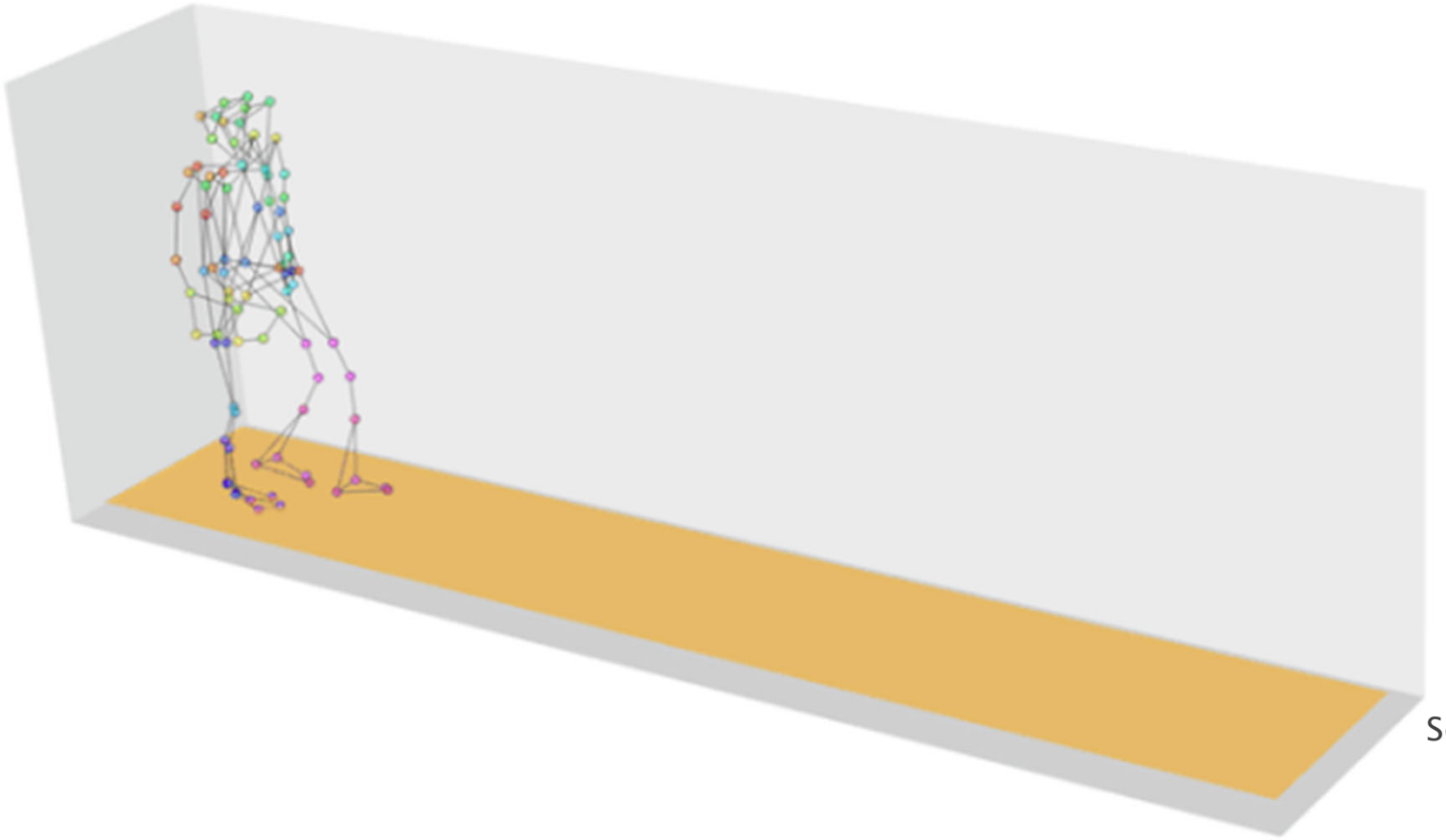


Figure 2. The two most common constraints in the literature are the Sakoe-Chiba Band and the Itakura Parallelogram

Comment 2: Applicable for Multivariate Time Series



Source [16]

Comment 3: DTW is not a proper metric

In mathematics, a metric or distance function defines a distance between each pair of point elements of a set. Let a, b and c be arbitrary point elements of a set X, then the distance d satisfies the following three axioms:

1. $d(a,b) = 0 \iff a = b$, *identity of indiscernibles*
2. $d(a,b) = d(b,a)$, *symmetry*
3. $d(a,b) \leq d(a,c) + d(c,b)$. *triangle inequality*

Source [17]

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