

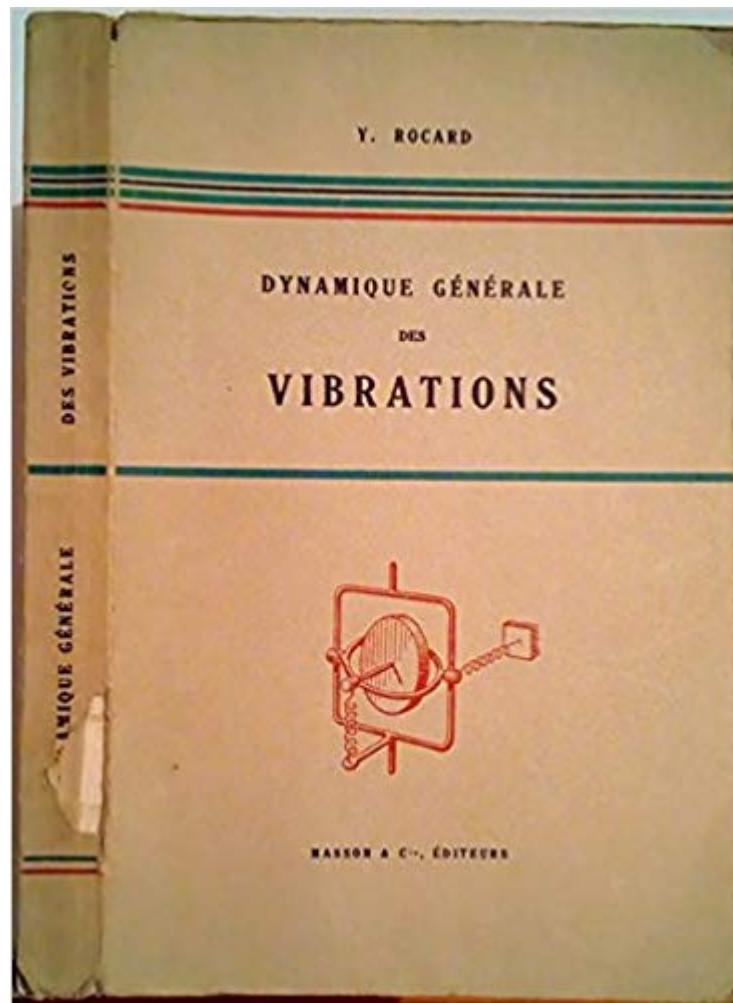


researchXchange

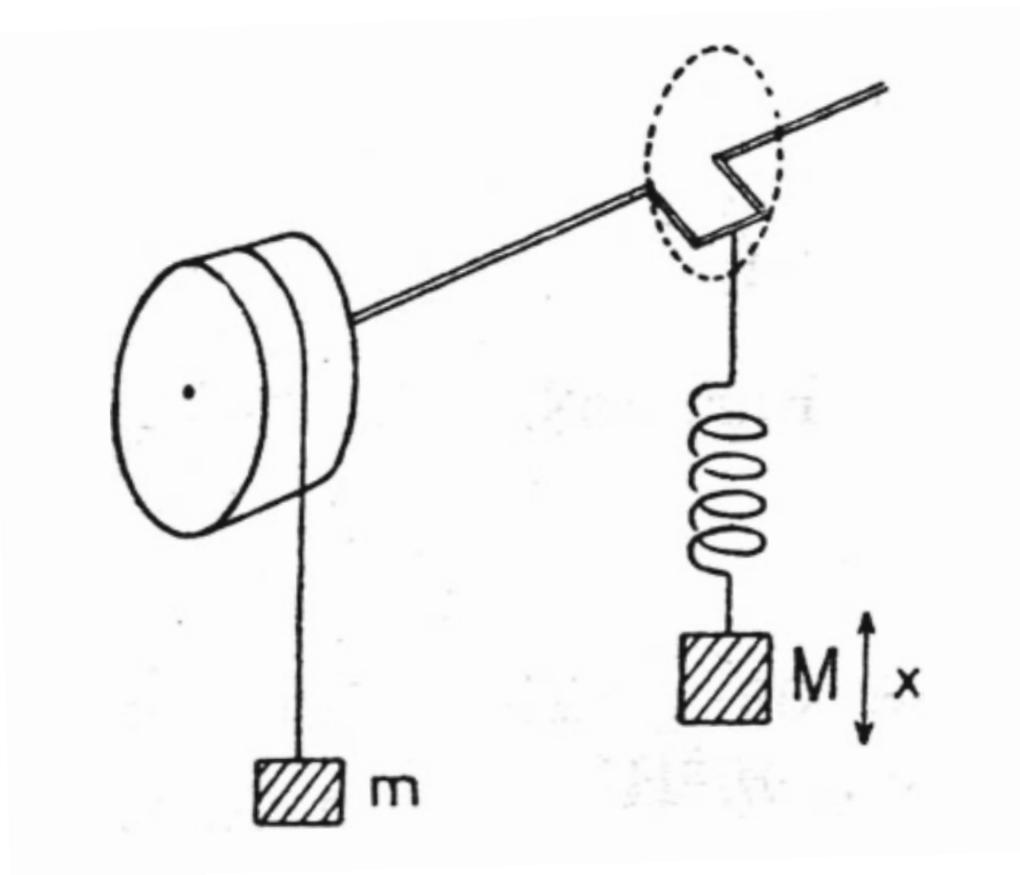
Welcome!

Tilted washboards and Chaotic Dynamics in a Vintage Rotating Roasting Spit

Prof. Dr. Julio Rodriguez



Y. ROCARD's 1943 book entitled:
"Dynamique Générale des Vibrations"



The Bouasse-Sarda's rotary spit drawn by Y. ROCARD in his 1943 book entitled:
“Dynamique Générale des Vibrations”

Presentation

I Setting

II Dynamics

III Numerical Simulations

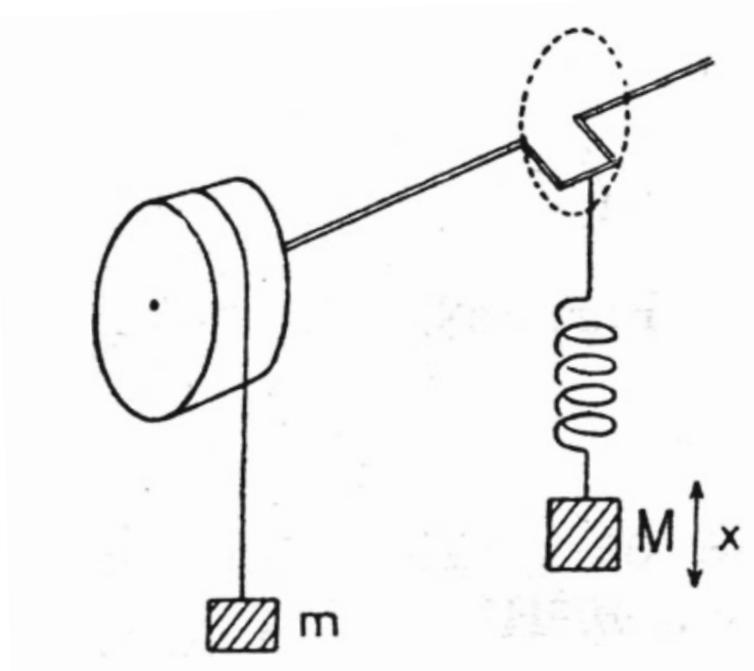
IV Route to chaotic evolution

V Antimonotonicity

VI Conclusions

Setting

- two degree of freedom: $\theta(t)$ (drum) and $x(t)$ (spring)
- 8 control parameters
 - drum: radius ρ , inertial moment \mathbf{I} , suspended mass m , friction h
 - spring: stiffness coefficient \mathbf{K} , suspended mass \mathbf{M} , friction f
 - rotating spite: crank/handel radius a



Setting

The total kinetic energy T and potential energy V of the system is

$$T(\dot{\theta}, \dot{x}) = (I + m\rho^2) \frac{\dot{\theta}^2}{2} + M \frac{\dot{x}^2}{2}$$

$$V(\theta, x) = -(mg\rho\theta + Mgx) + \frac{K(x - a\sin(\theta))^2}{2}$$

with g gravitation acceleration

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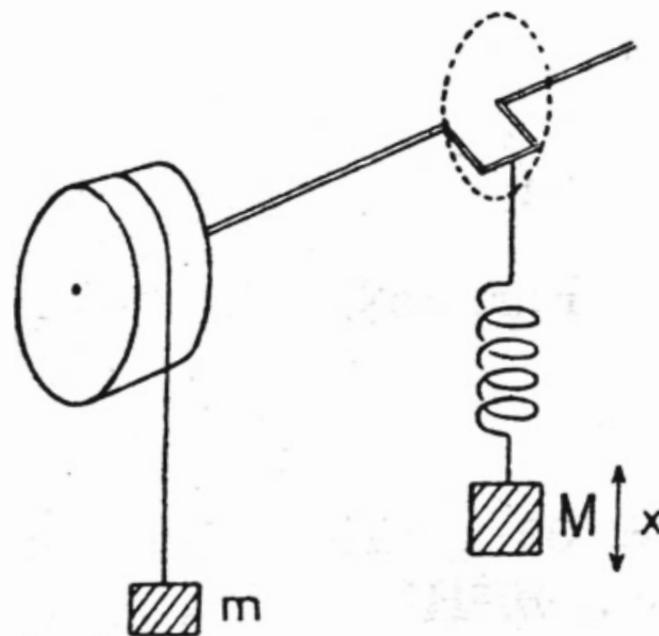
The Lagrangian

$$\mathcal{L} = T(\dot{\theta}, \dot{x}) - V(\theta, x)$$

Setting

$$(I + m\rho^2)\ddot{\theta}(t) + h\dot{\theta}(t) = mg\rho + K(x(t) - a \sin(\theta(t)))a \cos(\theta(t))$$

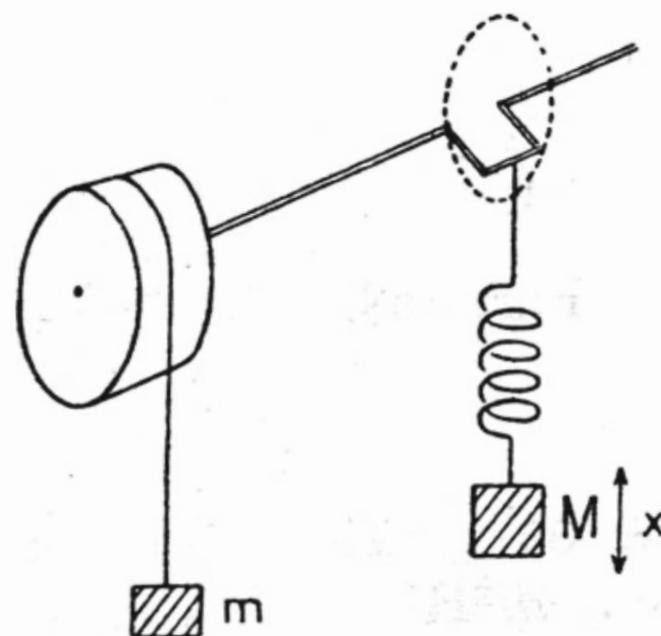
$$M\ddot{x}(t) + f\dot{x}(t) = Mg - K(x(t) - a \sin(\theta(t)))$$



Setting

$$(I + m\rho^2)\ddot{\theta} + h\dot{\theta} = mg\rho + K(x - a \sin(\theta))a \cos(\theta)$$

$$M\ddot{x} + f\dot{x} = Mg - K(x - a \sin(\theta))$$



Dynamics: Step (1)

Case: $a = 0$

$$J\ddot{\theta} + h\dot{\theta} = mg\rho$$

$$M\ddot{x} + f\dot{x} = Mg - Kx$$

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$$M\ddot{x} + f\dot{x} = Mg - Kx$$

with $J := I + m\rho^2$

Two decoupled linear ODEs - full solution is known and asymptotically

$$\lim_{t \rightarrow \infty} \theta(t) = \frac{mg\rho}{h}t + \check{\theta}$$

$$\lim_{t \rightarrow \infty} x(t) = \frac{Mg}{K}$$

Dynamics: Step (2)

Case: $a > 0$

The coupling on the drum

$$aK(x - a \sin(\theta)) \cos(\theta) \quad \text{is neglected}$$

Dynamics: Step (2)

Case: $a > 0$

$$J\ddot{\theta} + h\dot{\theta} = mg\rho$$

$$M\ddot{x} + f\dot{x} = Mg - K(x - a \sin(\theta))$$

Dynamics: Step (2)

Case: $a > 0$

$$J\ddot{\theta} + h\dot{\theta} = mg\rho$$

$$M\ddot{x} + f\dot{x} = Mg - K(x - a \sin(\theta))$$

Since

$$\lim_{t \rightarrow \infty} \theta(t) = \frac{mg\rho}{h}t + \check{\theta}$$

Dynamics: Step (2)

Case: $a > 0$

$$J\ddot{\theta} + h\dot{\theta} = mg\rho$$

$$M\ddot{x} + f\dot{x} = Mg - K(x - a \sin(\theta))$$

Since

$$\lim_{t \rightarrow \infty} \theta(t) = \frac{mg\rho}{h}t + \check{\theta}$$

Then (essentially)

$$M\ddot{x} + f\dot{x} = Mg - K\left(x - a \sin\left(\frac{mg\rho}{h}t + \check{\theta}\right)\right)$$

Dynamics: Step (2)

Case: $a > 0$

$$J\ddot{\theta} + h\dot{\theta} = mg\rho$$

$$M\ddot{x} + f\dot{x} = Mg - K(x - a \sin(\theta))$$

Since

$$\lim_{t \rightarrow \infty} \theta(t) = \frac{mg\rho}{h}t + \check{\theta}$$

Then (essentially)

$$M\ddot{x} + f\dot{x} = Mg - Kx + F \sin\left(\frac{mg\rho}{h}t + \check{\theta}\right)$$

Dynamics: Step (2)

Case: $a > 0$

$$J\ddot{\theta} + h\dot{\theta} = mg\rho$$

$$M\ddot{x} + f\dot{x} = Mg - K(x - a \sin(\theta))$$

Driven harmonic oscillators - full solution is known and asymptotically

$$\lim_{t \rightarrow \infty} \theta(t) = \frac{mg\rho}{h}t + \check{\theta}$$

$$\lim_{t \rightarrow \infty} x(t) = A \sin\left(\frac{mg\rho}{h}t + \check{\theta} + \varphi\right) + \frac{Mg}{K}$$

where A and $\tan(\varphi)$ are known

Dynamics: Step (3)

Case: $a > 0$

Change of variable: $x(t) = au(t) + \frac{Mg}{K}$

$$J\ddot{\theta} + h\dot{\theta} = \underbrace{g(Ma \cos(\theta) + m\rho)}_{H_a^{(1)}(\theta)} + \underbrace{Ka^2(u - \sin(\theta)) \cos(\theta)}_{H_a^{(2)}(\theta, u)}$$

$$Mu + fu = -K(u - \sin(\theta))$$

Remarks

- $H_a^{(1)}(\theta)$ does not dependent on u
- feedback through $H_a^{(2)}(\theta, u)$

Dynamics: Step (3)

Case: $a > 0$

Existence of stationary solution (i.e. fixed point)

$$\begin{aligned}(\theta^*, u^*) &= \left(\cos^{-1}\left(-\frac{m\rho}{\mathbf{M}a}\right), \sin\left(\cos^{-1}\left(-\frac{m\rho}{\mathbf{M}a}\right)\right) \right) \\ &= \left(\cos^{-1}\left(-\frac{m\rho}{\mathbf{M}a}\right), \sqrt{1 - \left(\frac{m\rho}{\mathbf{M}a}\right)^2} \right)\end{aligned}$$

Requirement for existence: $\mathbf{M}a \geq m\rho$

Linear Stability:

asymptotic stability guaranteed if f and h not simultaneously 0

Dynamics: Step (3)

Case: “small” $a > 0$

$H_a^{(2)}(\theta, u)$ is neglected

Dynamics: Step (3)

Case: “small” $a > 0$

$H_a^{(2)}(\theta, u)$ is neglected

$$J\ddot{\theta} + h\dot{\theta} = \underbrace{g(Ma \cos(\theta) + m\rho)}_{H_a^{(1)}(\theta)}$$

$$M\ddot{u} + f\dot{u} = -K(u - \sin(\theta))$$

Dynamics: Step (3)

Case: “small” $a > 0$

$H_a^{(2)}(\theta, u)$ is neglected

Change of variable: $\phi(\tau(t)) := \theta(t) + \frac{\pi}{2}$ and here $\tau(t) = \sqrt{\frac{\mathbf{M}a g}{\mathbf{J}}} t$

$$\phi'' + \beta \phi' + \sin(\phi) = \gamma$$

' denoting derivative with respect to τ , and $\beta := \frac{h}{\sqrt{\mathbf{J}\mathbf{M}a g}}$ and $\gamma := \frac{m\rho}{\mathbf{M}a}$

Dynamics: Step (3)

Case: “small” $a > 0$

Similar to particle inside a tilted washboard

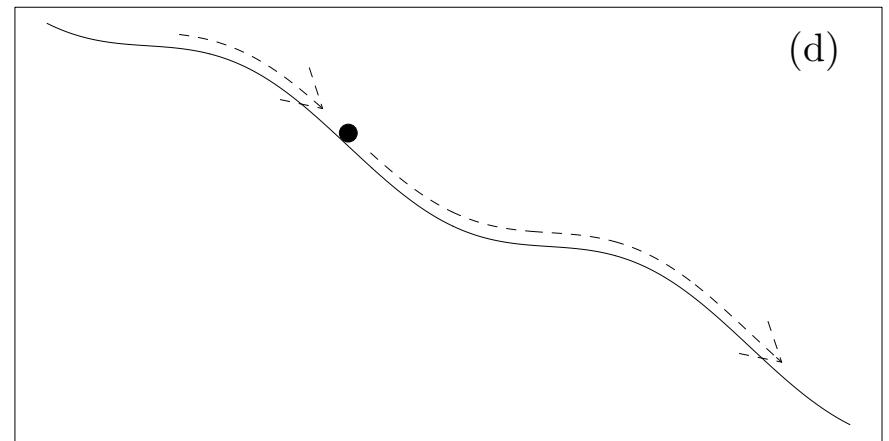
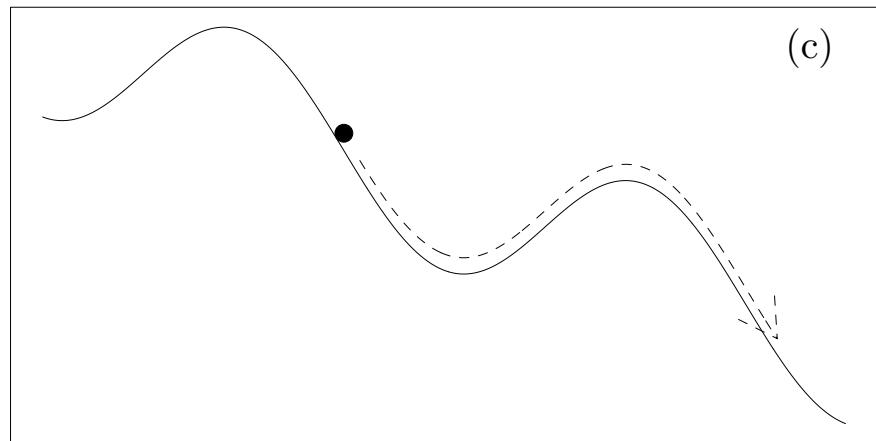
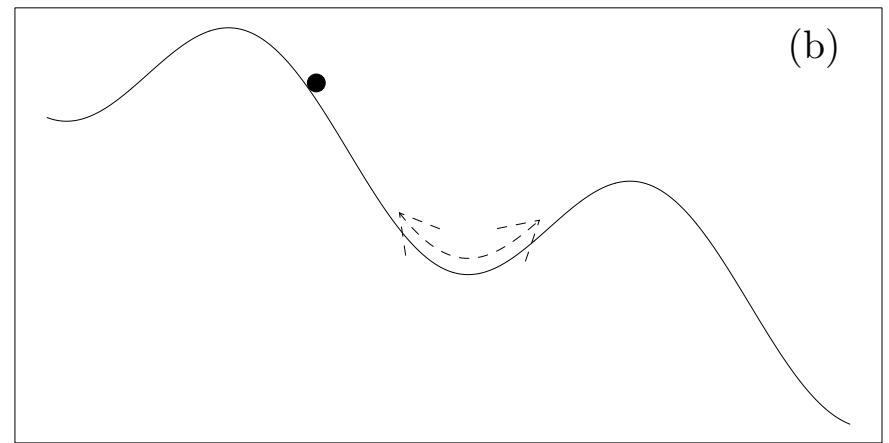
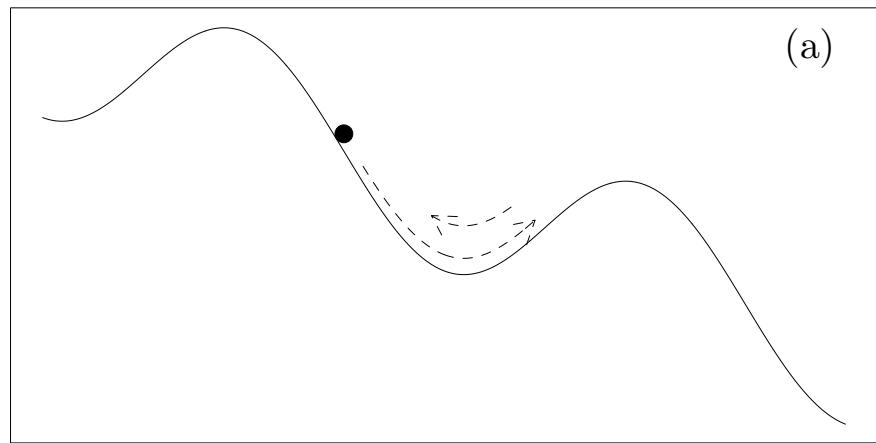
$$\phi'' + \beta\phi' + \sin(\phi) = \gamma$$

Dynamical system discussed in context of Josephson’s junctions dynamics

Equation (2) in P. Coulet, J. M Gillet, M. Monticelli and N. Vandenberghe, “*A damped pendulum forced with a constant torque*”. Am. J. of Phys. **73**(12), (2005), 1122-1128

Dynamics: Step (3)

Case: “small” $a > 0$

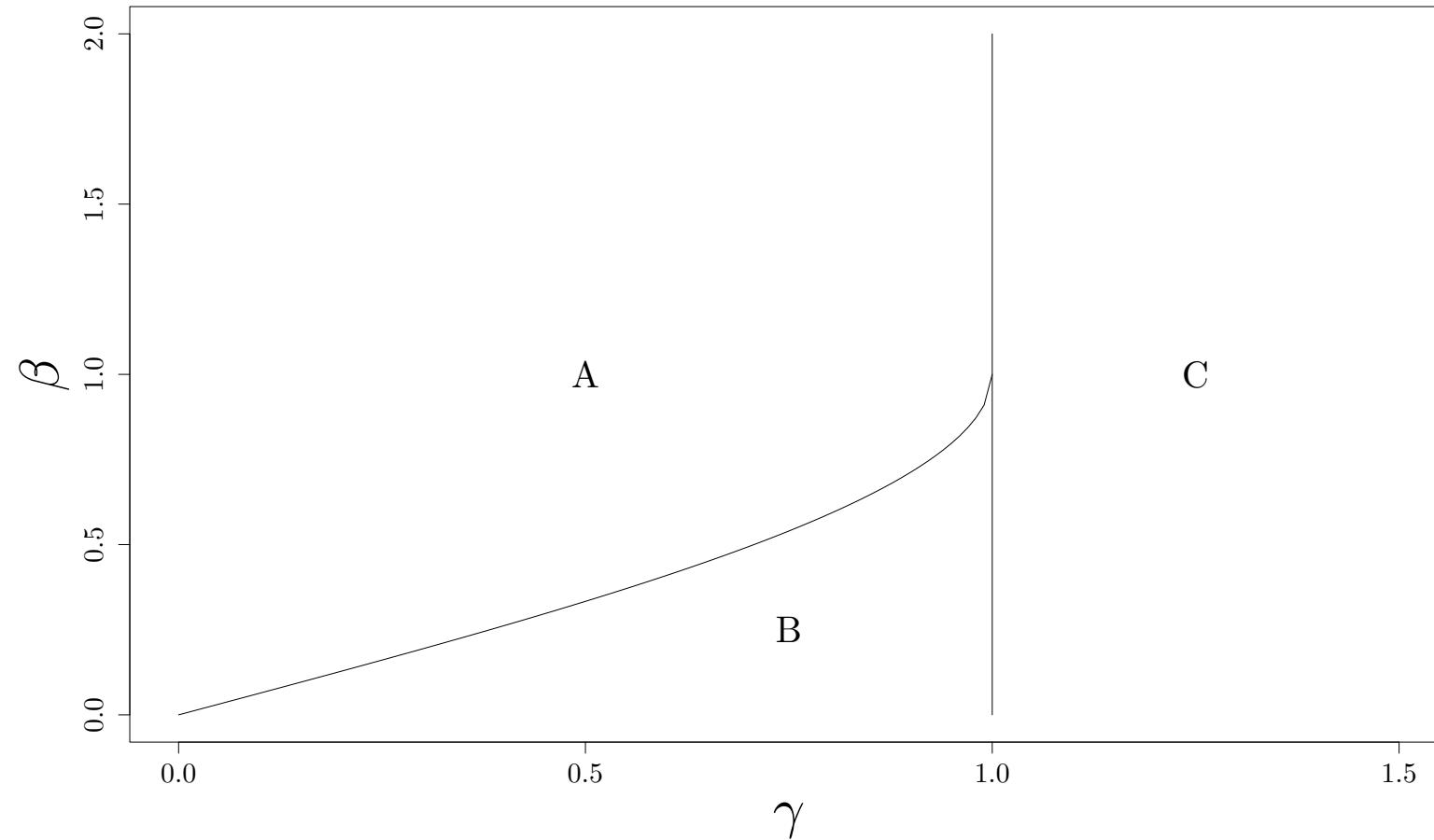


Evolution of a ball in a washboard potential:

- (a) stable stationary solution,
- (b) stable periodic solution,
- (c) stable stationary solution or stable periodic solution,
- (d) stable periodic solution (weak damping ($\beta < 1$) and strong external torque ($\gamma > 1$))

Dynamics: Step (3)

Case: “small” $a > 0$



Evolution regimes as a function of the damping β and the external torque γ

A = only stationary solution stable, B = both, stationary and periodic solutions, stable,
C = only periodic solution stable

Dynamics: Step (4)

Case: “large” $a > 0$

$H_a^{(1)}(\theta)$ and $H_a^{(2)}(\theta, u)$ are considered

Dynamics: Step (4)

Case: “large” $a > 0$

$H_a^{(1)}(\theta)$ and $H_a^{(2)}(\theta, u)$ are considered

As Y. ROCARD himself did - assume: $\theta(t) \cong \omega t$

Asymptotically with time, spring converges to an harmonic oscillatory state

$$\lim_{t \rightarrow \infty} u(t) = \frac{\frac{K}{M} \sin(\omega t + \varphi)}{\sqrt{(\frac{K}{M} - \omega^2)^2 + (\frac{f}{M})^2 \omega^2}} =: A \sin(\omega t + \varphi)$$

$$\text{with } \tan(\varphi) = \frac{-\frac{f}{M}\omega}{\frac{K}{M}-\omega^2}$$

Dynamics: Step (4)

Case: “large” $a > 0$

Question: what is the value of ω ?

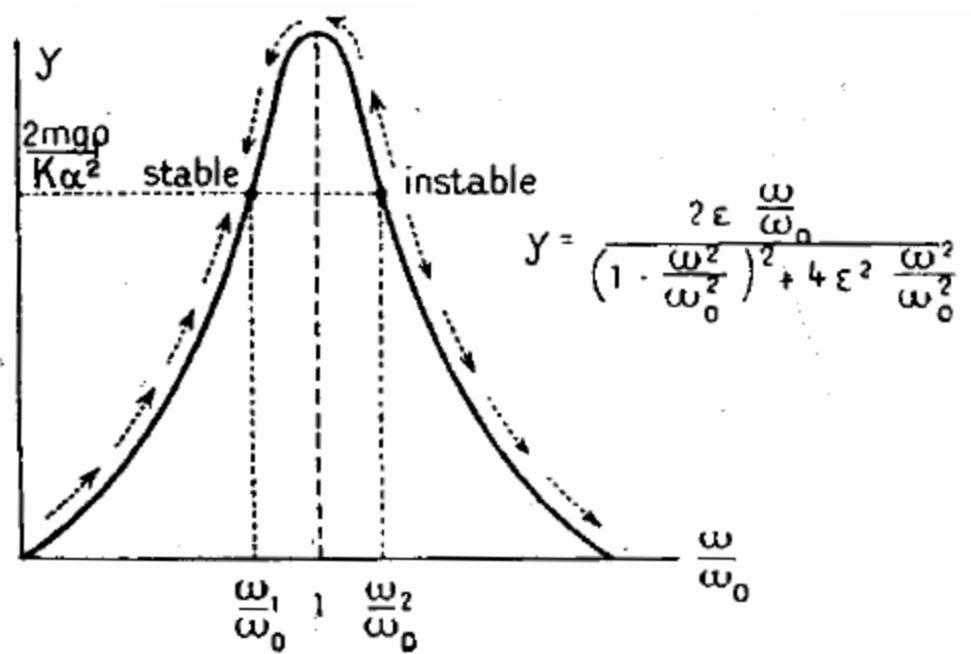
Dynamics: Step (4)

Case: “large” $a > 0$

Question: what is the value of ω ?

Y. ROCARD determines ω as:

$$\frac{2M^2mg\rho}{f(Ka)^2} = \frac{\omega}{(\frac{K}{M} - \omega^2)^2 + (\frac{f}{M})^2\omega^2}$$



Sketch drawn by Y. ROCARD in his 1943 book entitled: “*Dynamique Générale des Vibrations*”

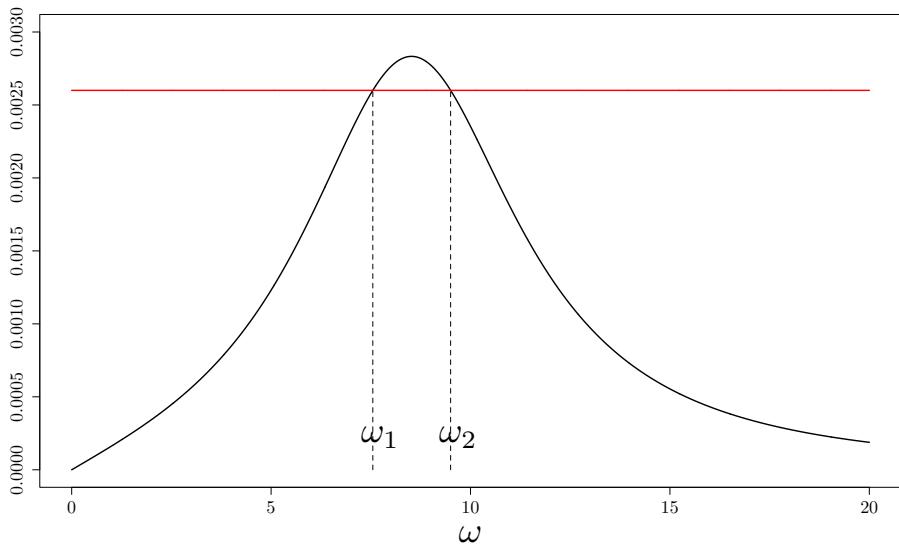
Dynamics: Step (4)

Case: “large” $a > 0$

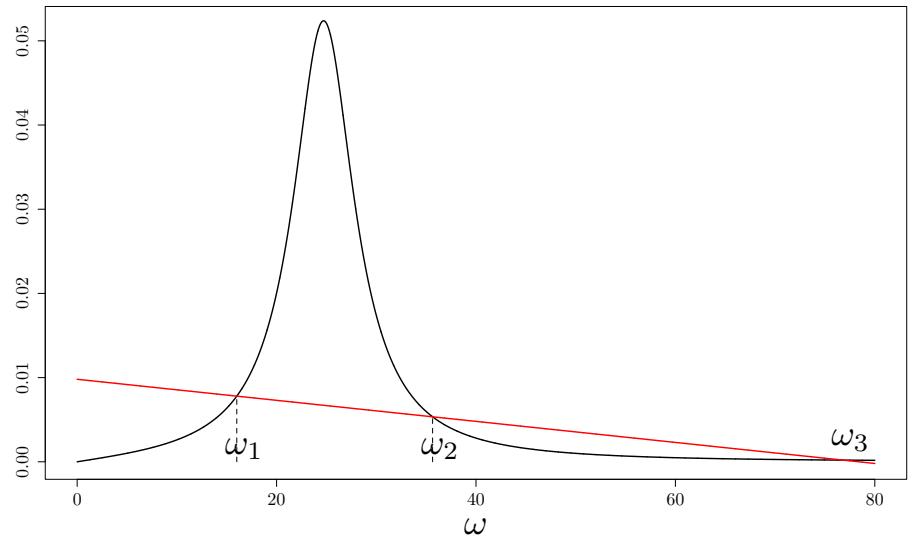
Question: what is the value of ω ?

For $h \geq 0$, ω determined as:

$$mg\rho - h\omega = \frac{f(Ka)^2}{2M^2} \left(\frac{\omega}{(\frac{K}{M} - \omega^2)^2 + (\frac{f}{M})^2 \omega^2} \right)$$



$h = 0$ up to two real roots



$h > 0$ up to three real roots

Numerical Simulations

- 8 control parameters

drum:

$$\text{radius } \rho = 0.01 \text{ [m]}$$

$$\text{inertial moment } I = 0.03 \text{ [kg][m]}^2$$

$$\text{suspended mass } m = 0.1 \text{ [kg]}$$

$$\text{friction } h = 0.000125$$

spring:

$$\text{stiffness coefficient } K = 12.5 \text{ [N]/[m]}$$

$$\text{suspended mass } M = 0.02 \text{ [kg]}$$

$$\text{friction } f = 0.15$$

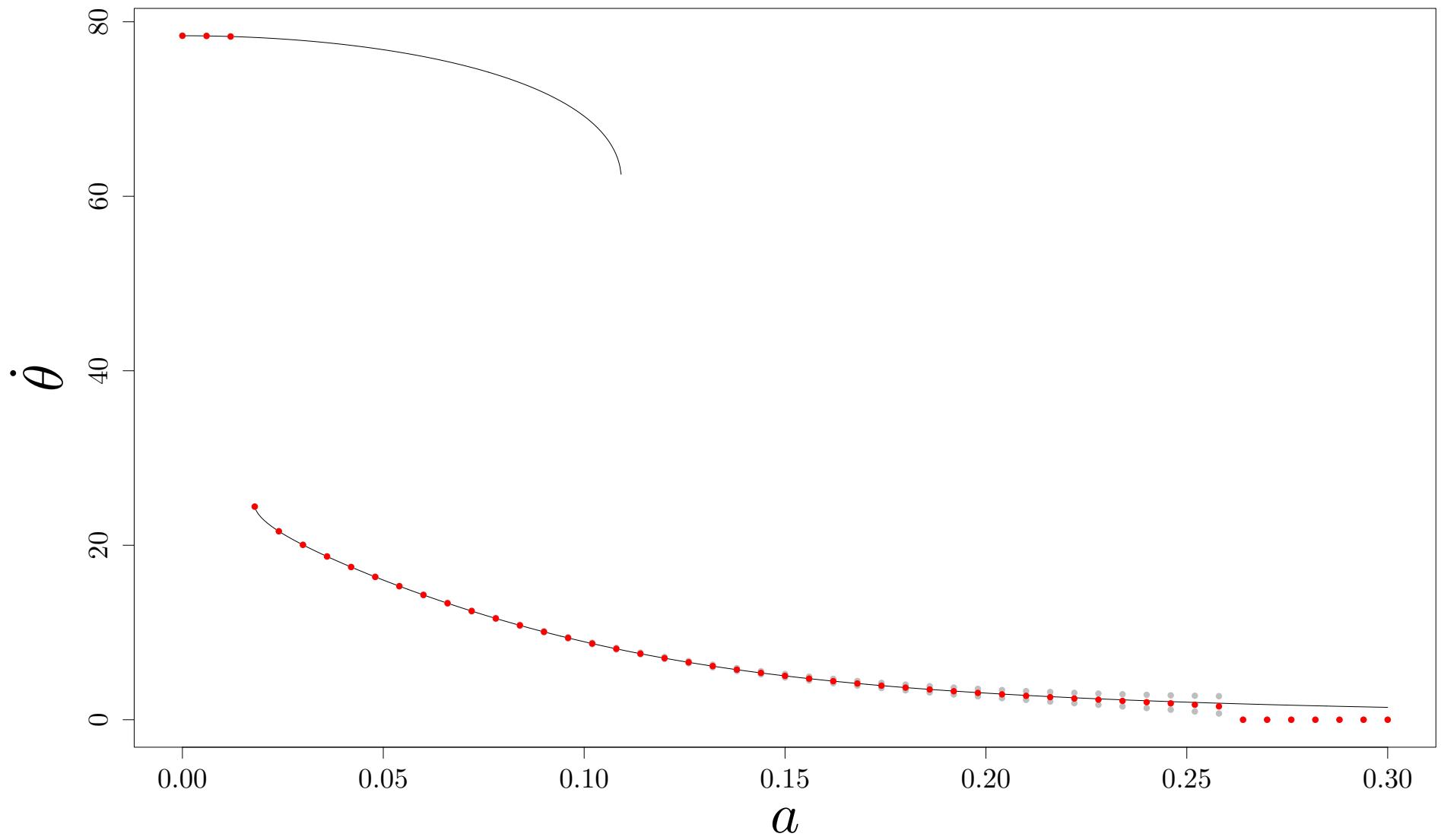
roataiting spite:

crank/handel radius $a \in [0, 0.3] \text{ [m]}$ (i.e. 51 equidistant points in this interval)

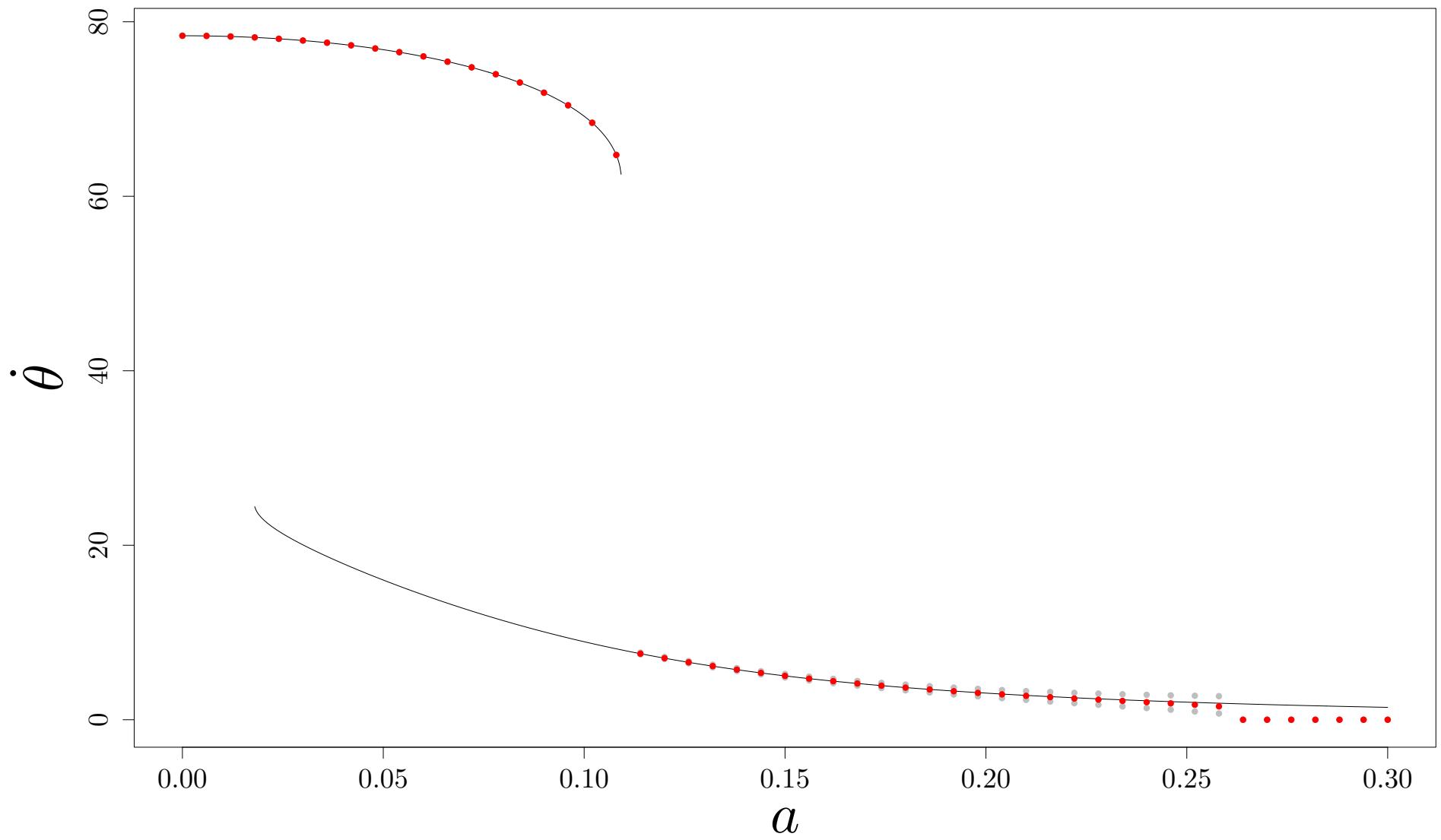
- + 1 more control parameter!

$$\text{gravitation acceleration: } g = 9.8 \text{ [m]/[s]}^2$$

Numerical Simulations



Numerical Simulations



Route to chaotic evolution

- 8 control parameters

drum:

$$\text{radius } \rho = 0.1 \text{ [m]}$$

$$\text{inertial moment } I = 9.24 \times 10^{-5} \text{ [kg][m]}^2$$

$$\text{suspended mass } m = 0.5 \text{ [kg]}$$

$$\text{friction } h = 0.03$$

spring:

$$\text{stiffness coefficient } K = 25 \text{ [N]/[m]}$$

$$\text{suspended mass } M = 0.5 \text{ [kg]}$$

$$\text{friction } f = 0.02$$

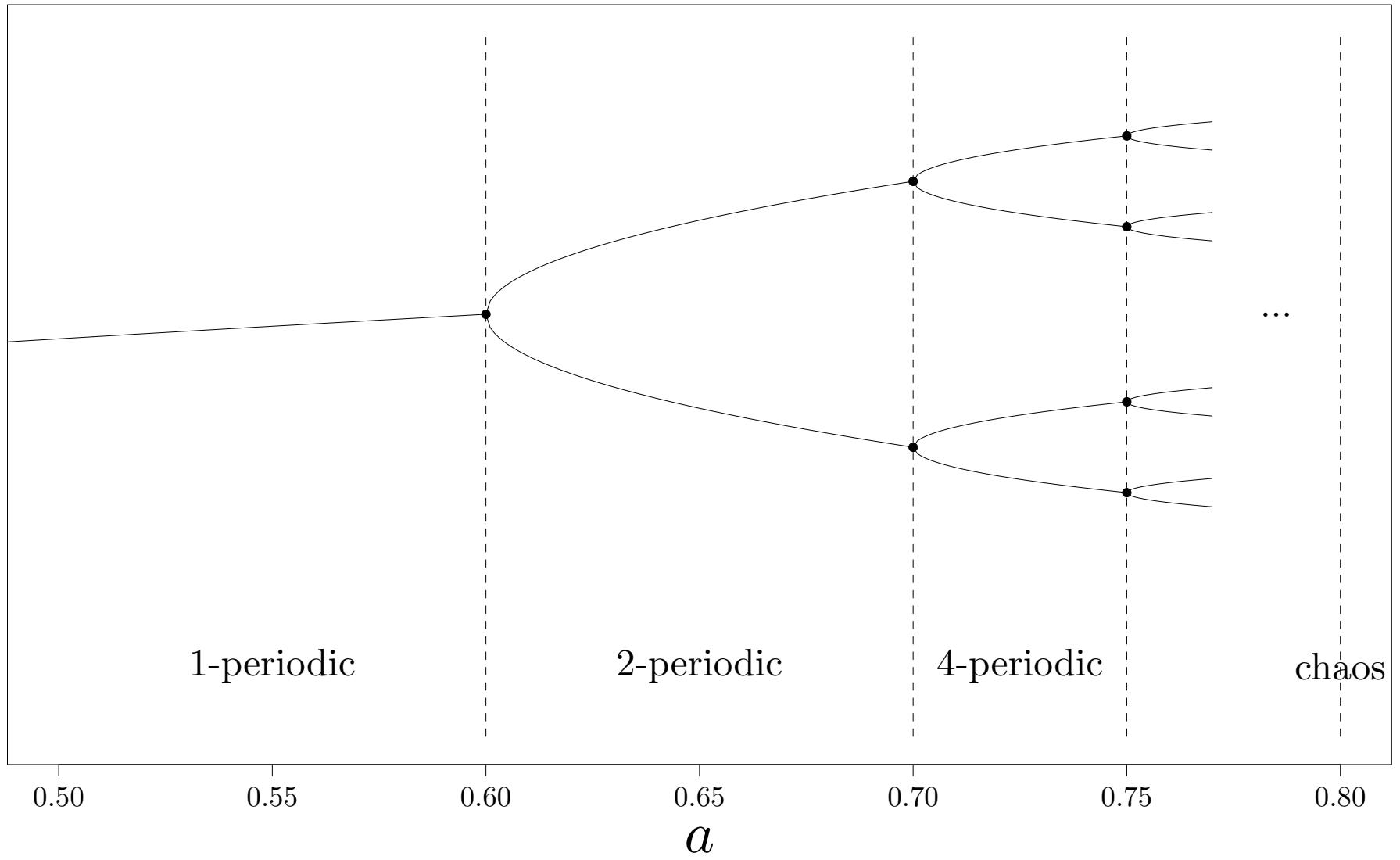
roataiting spite:

$$\text{crank/handel radius } a \in \{0.05, 0.07, 0.075, 0.08\} \text{ [m]}$$

- + 1 more control parameter!

$$\text{gravitation acceleration: } g = 9.8 \text{ [m]/[s]}^2$$

Route to chaotic evolution



Route to chaotic evolution

- 8 control parameters

drum:

$$\text{radius } \rho = 0.016 \text{ [m]}$$

$$\text{inertial moment } I = 0.0001 \text{ [kg][m]}^2$$

$$\text{suspended mass } m = 0.1 \text{ [kg]}$$

$$\text{friction } h = 0 \text{ (as in Y. ROCARD's book)}$$

spring:

$$\text{stiffness coefficient } K = 2.5 \text{ [N]/[m]}$$

$$\text{suspended mass } M = 0.03 \text{ [kg]}$$

$$\text{friction } f = 0.2$$

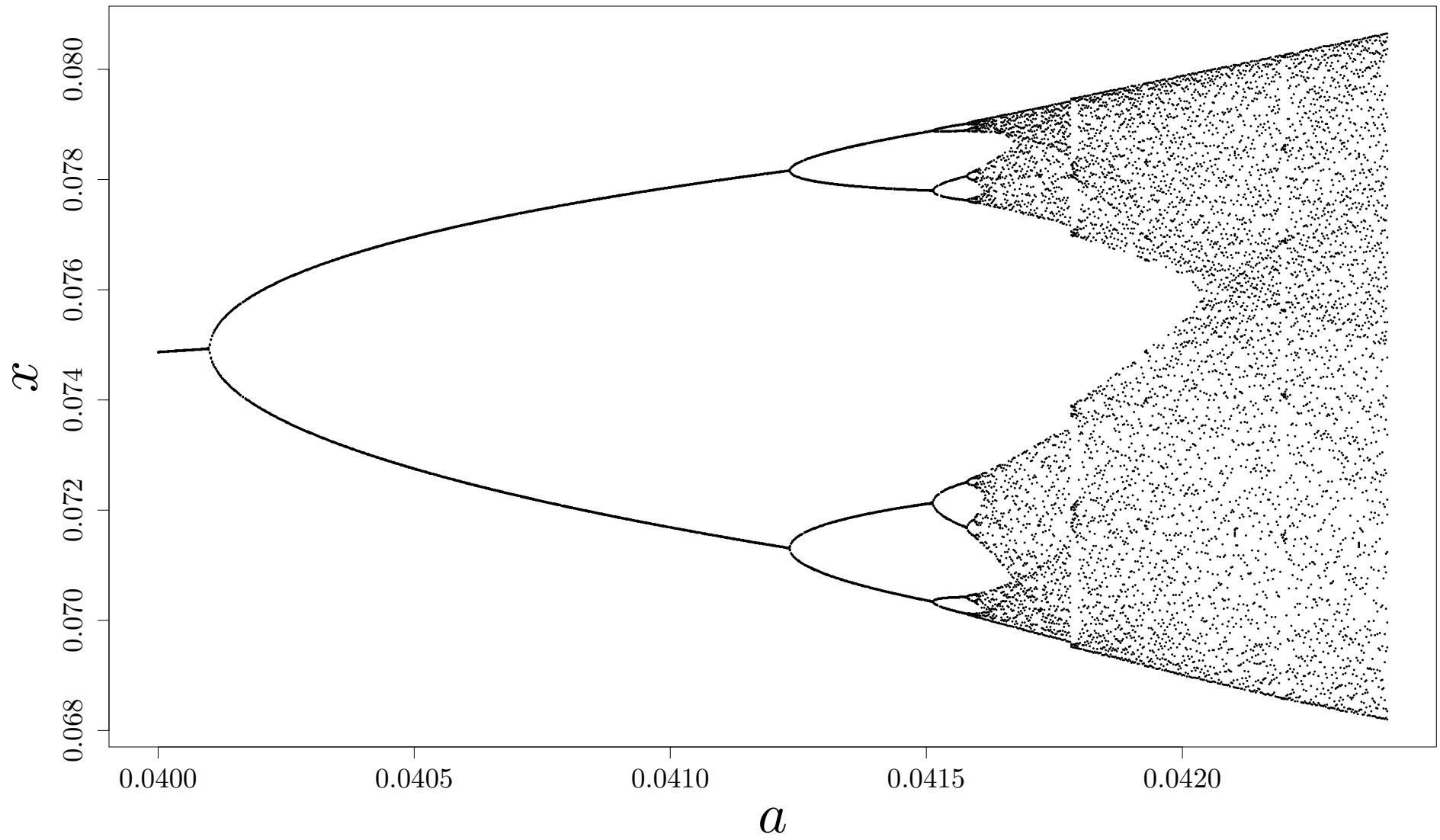
roataiting spite:

$$\text{crank/handel radius } a \in [0.04, 0.0424] \text{ [m]}$$

- + 1 more control parameter!

gravitation acceleration: $g = 1.625 \text{ [m}/[\text{s}]]^2$ (... on the moon!)

Route to chaotic evolution



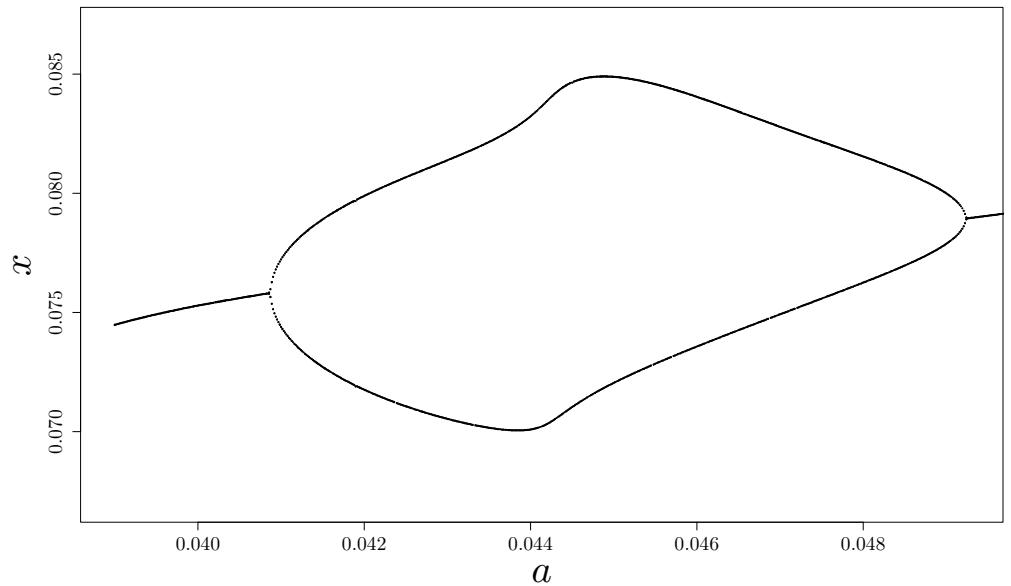
Route to chaotic evolution

Feigenbaum constant

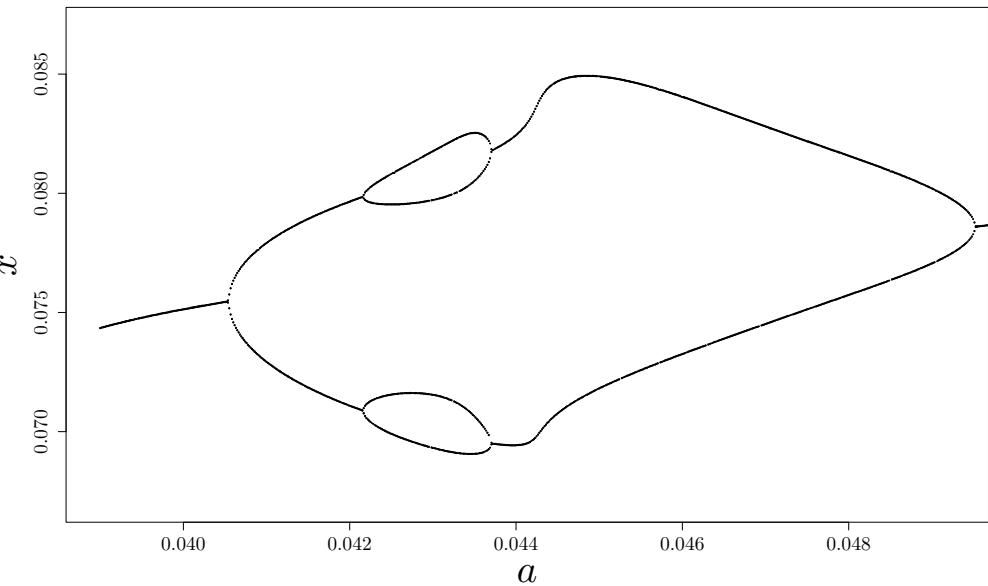
$$\lim_{n \rightarrow \infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} = 4.669201609\dots$$

| n | Period | Bifurcation parameter (a_n) | Ratio $\frac{a_{n-1}-a_{n-2}}{a_n-a_{n-1}}$ |
|-----|--------|---------------------------------|---|
| 1 | 2 | 0.04009785 | - |
| 2 | 4 | 0.04123185 | - |
| 3 | 8 | 0.04151235 | 4.042781 |
| 4 | 16 | 0.04157945 | 4.180328 |
| 5 | 32 | 0.04159425 | 4.533784 |
| 6 | 64 | 0.04159745 | 4.625000 |
| 7 | 128 | 0.04159815 | 4.571429 |

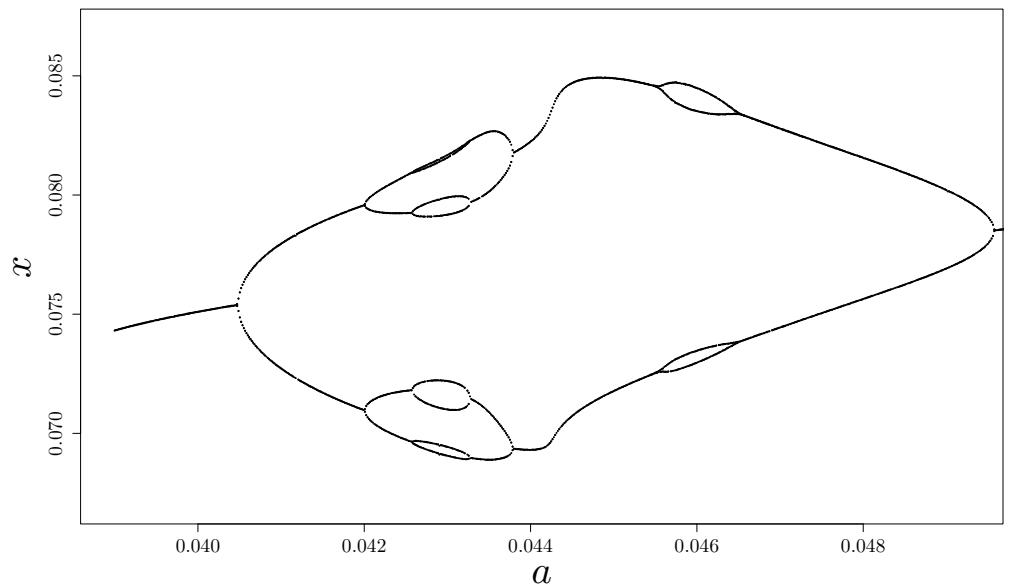
Antimonotonicity



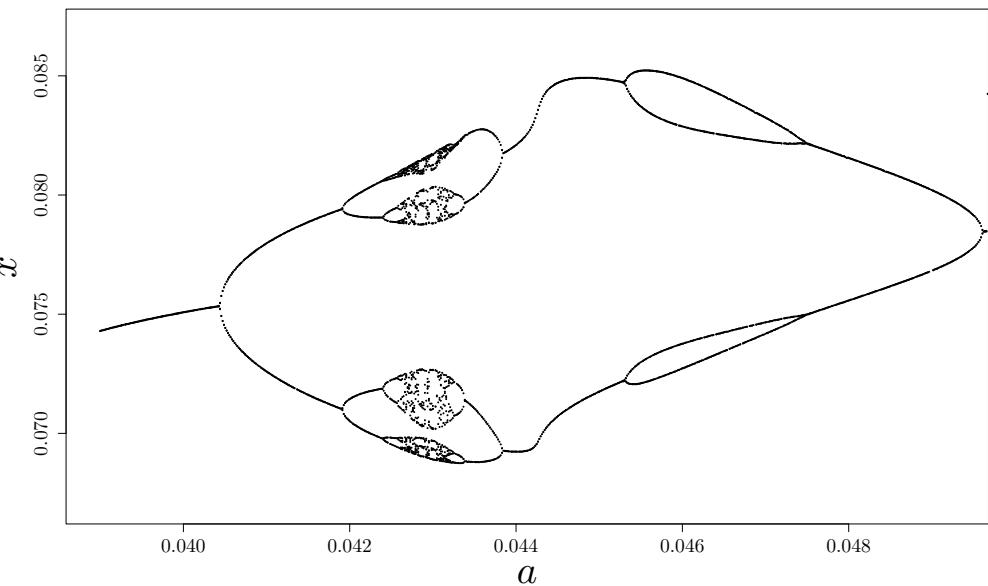
$$I = 0.000115$$



$$I = 0.000109$$



$$I = 0.0001078$$



$$I = 0.000107$$

Conclusions

I Y. ROCARD's remarkable intuition → very good approximation of the “consensual” frequency

“La physique, c'est toujours un petit peu faux” disait-il plaisamment

II Very rich dynamical system

III For “small” a , no chaotic dynamics

IV Effect of feedback (i.e. “large” a) → period-doubling bifurcation is observed

V Antimonotonicity is observed

References

- RODRIGUEZ, J., HONGLER, M.-O. How Chaotic Dynamics Drive a Vintage Grill-Room Spit. In *13th Chaotic Modeling and Simulation International Conference*. Springer International Publishing, 2021, 695-720.
- HONGLER, M.-O., RODRIGUEZ, J., Comme une bouteille à la mer... échouée au bord du Léman. To appear in *Editions Hermann*.

Next Meetings

Biel/Bienne
Quellgasse 21, Aula

14.10.22 Pegasus Spine, ein Gerät zur intelligenten Rückenbehandlung Jan Segessenmann, Wissenschaftlicher Mitarbeiter, Institute for Human Centered Engineering HuCE, BFH-TI

28.10.22 UAS-Thermaldaten zur Erkennung von Steinen auf landwirtschaftlichen Nutzflächen Prof. Florian Thürkow, Dozent, Fachbereich Wirtschaftsingenieurwesen, BFH-TI

11.11.22 The GNU Taler Payment System Prof. Dr. Christian Grothoff, Professor & CEO, Institute for Cybersecurity and Engineering ICE, BFH-TI & Taler Systems SA

25.11.22 Experimental heart rate variability characterization Lars Brockmann, Assistant, Institute for Human Centered Engineering HuCE, BFH-TI

09.12.22 Parylene-based encapsulation technology for wearable or implantable electronic devices Dr. Andreas Hogg, CEO, Coat-X AG, La Chaux-de-Fonds

13.01.23 Care@Home mit technischer Unterstützung Prof. Dr. Sang-II Kim, Professor, Institute for Medical Informatics I4MI, BFH-TI

Burgdorf/Berthoud
Pestalozzistrasse 20, E 013

21.10.22 Robot Task Model and Notation Congyu Zhang Sprenger, Wissenschaftliche Mitarbeiterin, Institute for Intelligent Industrial Systems I3S, BFH-TI

04.11.22 Data Science for Startups im ZID, Bernapark, Stettlen, Bern Prof. Dr. Erik Graf, Dozent, Institute for Data Applications and Security IDAS, BFH-TI

18.11.22 Flexible programming of Industrial Robots for Agile Production environments Laurent Cavazzana, Research scientist, Institute for Intelligent Industrial Systems I3S, BFH-TI

02.12.22 Wie gefährlich ist ein Unfall mit einem Cabriolet? Prof. Raphael Murri, Institutsleiter IEM, Institut für Energie- und Mobilitätsforschung IEM, BFH-TI

16.12.22 Systemtechnologie für die Mikrobearbeitung mit Hochleistungs-UKP-Lasern Prof. Dr. Beat Neuenschwander, Institutsleiter ALPS, Institute for Applied Laser, Photonics and Surface Technologies ALPS, BFH-TI